Solving a Practical Multi Depot Vehicle Scheduling Problem with Pickup and Delivery

Thesis Submitted to
The Superior College, Lahore

In Partial Fulfillment of the
Requirement for the Degree of

Master of Sciences in Supply Chain Management

By
Muhammad Furqan
Roll No. MSSC-F16-003
Session: 2016-2018

The Superior College (Faculty of Management Sciences), Lahore

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Approved By:

________________________	________________________
Dr. Khyzer Bin Dost	Dr. Raza Saleem Khan
Program Manager, Research Degrees	Research Supervisor
________________________	________________________
Dr.	
External Examiner	Additional Controller Examination

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Faculty of Management Sciences, Lahore, Pakistan

DECLARATION TO BE FILLED BY THE STUDENT AT THE TIME OF SUBMISSION OF THESIS TO THE SUPERVISOR AND/OR FOR EXTERNAL EVALUATION

Section 1: Particular of the Student		
1.1	Full Name	Muhammad Furqan
1.2	Father's Name	Shahid Aziz
1.3	Roll. Number	MSSC-F16-003
1.4	Program	Master of Sciences in Supply Chain

Section 2: Particular of the Thesis		
2.1	Title	Solving a Practical Multi Depot Vehicle Scheduling Problem with Pickup and delivery
2.2	Supervisor's Name	Dr. Raza Saleem Khan
2.3	Date of Completion	March 10, 2020

The Superior College

Faculty of Management Sciences, Lahore, Pakistan

SUPERVISOR'S CERTIFICATION

THESIS SUBMITTED BY A STUDENT

Section 1: Particulars of the Supervisor		
1.1	Full Name	Muhammad Furqan
1.2	Address	The Superior College, Faculty of Management Sciences, Lahore.

Section 2: Particulars of the Student		
2.1	Full Name	Muhammad Furqan
2.2	Father's Name	Shahid Aziz
2.3	Registration Number	MSSC-F16-003
2.4	Program	Master of Sciences in Supply Chain

Section 3: Particulars of the Thesis		
3.1	Title	Solving a Practical Multi Depot Vehicle Scheduling Problem with Pickup and delivery
3.2	Date of Completion	March 10, 2020

I certify that:

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DEDICATION

This dissertation is wholeheartedly dedicated to my Almighty Allah, who has given me the mental strength, and the abilities that helped me to accomplish this. Moreover, this work is dedicated to my supervisor, family and friends who encouraged me, trusted me, and prayed for me throughout the project.

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Name of Research Scholar

Muhammad Furqan

ABSTRACT

Background: The vehicle scheduling problem (VSP) is considered one of the major issues in logistics. Due to its inherent complexity, it's difficult to solve the problem even when there is only one depot. To solve the multi-depot vehicle scheduling problem (MDVSP), it becomes even more complex.

Problem: Route planning is considered a major issue in practical life because it directly impacts the cost, travel time, distance traveled, and fuel consumption. In this research, we are dealing with a vehicle scheduling problem which is linked with the multi depot, pickup, and delivery problem along with the characteristics of Dial a Ride Problem (DARP).

Purpose: The main purpose of this research is to find out the solution through which we can keep the distance cost at its minimum.

Methodology: The research is based on a case study approach where linear programming has been used for real-life and randomly generated data.

Findings: The contribution of this research will be in the form of a model that will combine different aspects of the vehicle routing problem. Further, it will be beneficial for the company to reduce the distance cost.

Key Words: Vehicle Scheduling/Routing Problem, Multi-Depot, Pickup and Delivery problem, Dial-a-Ride Problem

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List of Abbreviations

CVRP	Capacitated Vehicle Routing Problem
DARP	Dial-a-Ride Problem
LP	Linear Programming
MDVRP/MDVSP	Multi Depot Vehicle Routing/Scheduling Problem
MDVRPPD/MDVSPPD	Multi Depot Vehicle Routing/Scheduling with Pickup and Delivery
MIP	Mixed Integer Programming
NP-Hard	Non-deterministic Polynomial-time Hardness
TSP	Travelling Salesman Problem
VRP/VSP	Vehicle Routing/Scheduling Problem
VRPB	Vehicle Routing Problem with Backhauls
VRPPD	Vehicle Routing Problem with Pickup and Delivery
VRPTW	Vehicle Routing Problem with Time Windows

Chapter No: 1 Introduction

This chapter starts by providing the detail of the vehicle scheduling problem or vehicle routing problem, its typical constraints, and objectives. Afterward, the focus switches to specific variants of the vehicle scheduling problem being faced by a UK-based distribution company that motivated me to carry out this study. This section concludes by providing information related to overall objectives and significance (i.e. practical application) of this research.

1.1 Background of the Study

According to Kherbach & Mocan (2016), Logistics is considered as an important part of the supply chain management. "*It deals with making plans for better storages, efficient and effective flow of products, services, and information from one point to another for the fulfillment of customer requirements*". According to Toth & Vigo (2002), the cost to transport goods from point of origination to point of consumption is typically equivalent to 10 to 20% of the total cost of a product. Increased transportation cost, time and distance have raised the focus of management on logistics-related decisions in companies (Kherbach & Mocan, 2016). The efficient management of Logistics activities and associated costs play a crucial role in the success of a product or a company (Palmer, 1998).

One of the important logistics activities is known as the vehicle routing problem (VRP). Dantzig and Ramser (1959), firstly introduced and solved the problem. In this problem, "*optimal routes are designed for the pickup and delivery of the products from one or multiple depots, geographically distributed customers and cities subject to some constraints*" or "*this problem is stated for the determination of optimal routes. These routes are based on customers' orders for* *goods, which are done within a set time, through the distribution and movement of goods by a set of vehicles"* (Paolo Toth, 2002)

VRP problems are usually solved by representing the road network through graph theory. In that graph, customers (manufacturers, suppliers, retailers, distribution centers or final consumers) and warehouses/depots are represented as nodes; and roads between these nodes are represented as links that connect two nodes. For every link, there is an associated cost that could represent either distance or travel time or cost between these two nodes. The customers located at the nodes have some typical characteristics which are the following:

- Customers' demand which includes pickup and delivery process for various products at customer nodes.
- The time required to serve the customer's nodes. This time includes travel time and service time for the vehicles.
- Vehicles are required to serve the customers' nodes. These vehicles may vary from each other depends on the customer's demand like vehicles with different capacity requirements (Paolo Toth, 2002).

For each graph, nodes have some starting and ending points, normally known as warehouses/depots which serve the customers nodes. These depots also have some characteristics like, parking capacity and types of vehicles at the depot, product's quantities and types with which depots are dealing. While in some real-world cases, customer's nodes have priority distribution at the depots where each vehicle must go back to its home depot (where it started from) (Paolo Toth, 2002). In such type of cases, it is tough to solve VRP, when each vehicle or customer's request is linked with a different depot or more than one depot.

According to Toth & Vigo (2002), in graph theory for VRP, customers, and warehouses/depots nodes are connected through the roads (also known as links) and their networks. Where vehicles mainly use these roads and their network for the distribution of goods at the customer nodes. These vehicles may have various sizes and composition which can be considered as fixed or may vary as per customer's request. Main features for the vehicles are the following:

- Vehicles' home depot, with a possibility to end its service at its home depot or other than its home depot.
- Vehicles' capacity, capacity can be determined in the form of pallets, volume and maximum load that can be loaded on the vehicle.
- Vehicles' subdivision, this subdivision is based on different sections having different products and capacity.
- Tools available for loading and unloading of vehicles
- Links and their subsets (parts of the complete road network) which can be pass-through the vehicles.
- The cost linked with vehicle utilization that can be in the form of distance, time and route, etc.

While traveling on the roads by vehicles, it is necessary to take care of the constraints which are linked with the drivers who are using these vehicles. Constraints like working time of a day, break duration during service time, the maximum period for driving and overtime (Paolo Toth, 2002).

As per Toth & Vigo (2002), it is necessary for each route that it must satisfy the constraints. These constraints are linked with products which need to be transported, service levels, customer request, and vehicle requirements. Following are the typical constraints for VRP:

- The vehicle load should not be exceeded than its capacity on each route.
- Customer must be served in a route which may have a pickup request or delivery request or both at the same time.
- Customers should be served within described time windows on a route.
- Customers must be served under the working period of drivers.
- Precedence constraint (constraints, link the tasks together) used for the orders that need to be served to those customers who are located on the same route.
- Another precedence constraint is linked with the pickup and delivery problems in which a vehicle needs to perform both pickup and delivery for the requested customers. In this constraint, pickup needs to be delivered to the corresponding customer. Such type of constraint is known for pickup and delivery problems (Xu & Chen, 2001) (Bergvinsdottir, Larsen, & Jorgensen, 2004).
- One precedence constraint is related to customer’s different types in which customers need to be served as per their fixed orders in a route. This condition represents the VRP with backhauls (Xu & Chen, 2001) (Paolo Toth, 2002).

As discussed by Toth & Vigo (2002), for the estimation of global cost and to check the working of the constraints, it is necessary to have the cost and travel time information between each node. In actual cases, the road graph between nodes is sparse which normally converted into a complete graph (each node is connected through a link). This is why, a complete graph is used in most problems of vehicle routing which is based on the cost and travel time information (M. W. P. Savelsbergh, 1995) (Paolo Toth, 2002). Based on the graph theory, typical objectives in VRP are given below:

- Cost minimization, based on the distance or the travel time or the fixed cost accompanying the vehicles.
- Vehicles or the drivers' minimization which are serving the customers.
- Routes balancing for load and travel time.
- Reduction in penalties that are linked with unserved or partially served customers.

In VRP, few types of research and applications have been made in the past in which one vehicle can serve multiple routes in a given time. These past researches and applications have been done on the bases of two types of demand linked with the customer nodes. One type is known for the static demand for which all the demand data is already known for the customer nodes. The second type of demand is known as time-dependent dynamic demand. In this type fewer data about customers' demand, cost and travel time is available or this data is continuously changing. (Lars M. Hvattum, 2006).

Depending on the road network data, objectives and constraints, the solution for VRP is only possible when the objectives linked with the problem are achieved. Constraints for the problem are operational. But in some real-world cases, it is not possible or it's tough to achieve all the objectives. In such cases, customer pickup or delivery requests can be minimized or it can be left without serving. In such scenarios, penalties or different priorities are associated with the routes (Paolo Toth, 2002).

In recent times vehicle routing/scheduling problem has become a major consideration for those companies which are dealing with the distribution or transportation sector (Kherbach & Mocan, 2016). As the problem, affects the vehicle scheduling/routing due to which companies are facing issues with time and cost. This is why, the planners of those companies need to have a strong understanding of the VRP and its variants, to have optimal solutions. In this problem, services of the companies are linked with the pickup and delivery of the orders at customer nodes (retail, distribution or consumer point) through the vehicles originate and terminate at the central depot (Desrochers, Desrosiers, & Solomon, 1992). It is known as the extension of VRP called VRP with pickup and delivery.

In VRP with Pickup and delivery (VRPPD), most researchers assume that customer nodes can be segregated into two types, where vehicles not only have to deliver the products to the line haul customers (*which are receiving products*) but also need to pick up the products from backhaul customers (*which are sending products*) (Parida Jewpanya, 2016). VRP with pickup and delivery (VRPPD) has the following variants:

- ***Pickup First, delivery second*****:** In this variant, vehicles made all the pickup first from the backhaul customer nodes and then deliver it to the line haul customer nodes. It is known as pickup first, delivery second. During the execution of the variant, it often becomes complex to reorganize the delivery with a new pickup on the same vehicle. It may cause pallet shuffling or re-arrangement, which cause delays (Paolo Toth, 2002).
- ***Mixed pickup and delivery.*** In this variant, the vehicle needs to serve line haul and backhaul customers in any sequence on a route. It is known as mixed pickup and delivery.

Mixed pickup and delivery and pickup first, delivery second variants are referred equally for the VRP with backhauling customers (Paolo Toth, 2002).

- ***Simultaneously Pickup and delivery.*** In this variant of VRP, the customer can deliver and receive the product simultaneously. While Mixed pickup and delivery can also be used in the same framework with the simultaneously pickup and delivery by keeping line haul or backhaul customer at nil (GÁBOR NAGY, 2005).

The vehicle routing problem has been extended over a period of time. Where another extension of VRP is known as the Dial and ride problem (DARP) "*which is based on the vehicles' routes designing and user scheduling that helps to determine the pickup and delivery orders among its starting and endpoint"* (Paolo Toth, 2002). In DARP, commonly, one user may have more than one order during the same day. These orders can be inbound or outbound. *Inbound orders- belongs to a destination point from home* while *outbound orders-belongs to a reverse trip.* Common examples which are taken for DARP is known as transportation service for aged and disabled patients. DARP is commonly used by western countries in the response to increasing demand and renovating the current systems. Where this concept played a vital role in the development of ambulatory services (Kiechle, Doerner, Gendreau, & Hartl, 2007).

From the modeling perspective, DARP generalizes several VRP problems like vehicle routing problems with time windows (VRPTW) and vehicle routing problems with pickup and delivery (VRPPD). What makes DARP different from the other routing problems is the human view. It provides assistance to transport passengers or users and minimize their inconveniences by keeping operating costs at its minimum (Bergvinsdottir, Larsen, & Jorgensen, 2004). Furthermore, in DARP vehicle capacity for the demand is used commonly while in VRPPD it is often laid off, a common example is letters or parcels collection problem (Paolo Toth, 2002).

DARP is based on static or dynamic demand method. In the static demand method, all the orders demand is known (Bergvinsdottir, Larsen, & Jorgensen, 2004). In a dynamic demand method, orders demand revealed during the whole day. This demand hardly occurs because in most cases demand is known (Lois & Ziliaskopoulos, 2016). Where all the vehicles' routes need to be adjusted in real-time to fulfill customer demand.

Most cases of DARP have been done through with the homogeneous vehicles and a single depot. But in actual, there may some different cases which are based on heterogeneous vehicles and multiple depots. For example, ambulatory services in which some vehicles only carry wheelchairs or some vehicles that only carry the ambulatory patients or some vehicles may serve both types of orders (Bergvinsdottir, Larsen, & Jorgensen, 2004) (Kiechle, Doerner, Gendreau, & Hartl, 2007). Similarly, in some problems, it is necessary to know about the size and composition of vehicles. In this way, the maximum number of orders can be served (Jean-François Cordeau, 2002).

Normally, during pickup and delivery of the orders, a set of routes designed by which vehicles serve the orders (M. W. P. Savelsbergh, 1995). These orders can be in any form, it can be in the form of raw material, wastes, finished product or it could be in the form of chilled food. However, when the transportation of chilled food orders is required, this issue becomes more sensitive. (Rabbania, Farshbaf-Geranmayehb, & Haghjooc, 2016). This thing needs different temperatures that could be frozen, chilled and ambient. In the case of frozen, the temperature would be (-16-25°C), for chilled (0-5°C) and ambient (more than 5°C). For this, the distribution or transportation companies design cold chain vehicles. Which helps to maintain the temperature as per product requirements and save food from deterioration during pickup and delivery (Zhang & Chen, 2014). This is why travel time is also a factor that could impact the quality of chilled food and increase the cost for the company. Therefore, the vehicles need to make on-time delivery, it will not only save the food but also minimize the cost for the company. (Chaug-Ing Hsu, 2007).

One of the major areas of concern is the same for the UK retail sector company NFT. It is providing the distribution services of chilled food. The company is also dealing with other services like full load transportation services or retail distribution services. It is working with both channels primary and secondary. *The primary channel belongs from the manufacturer to the retailer's distribution center* while *the secondary channel is from the retailer's distribution center to the customer destination.* The company has both backhaul customers which are manufacturers (supplier nodes) and line hauls customers which are delivery customers (distribution centers) (Paolo Toth, 2002). For which, the vehicle will pick up the product orders first from suppliers' nodes and then deliver it to the distribution centers nodes. The company has its expertise in chilled food products' distribution by managing the different temperature requirements. The company is managing the products at different temperature levels like chilled (0-5°C), frozen (-16-25°C) and ambient (considered as normal or more than 5°C).

The company is managing all its processes of distribution and transportation for products through multiple depots and the modern vehicles system. Where the company is using articulated vehicles (based on 44 tons or 26 pallets capacity). For the pickup and delivery process, distribution centers make an order as per their requirements. After which the company planning team coordinates with its manufacturers and they pack the orders in the form of pallets. Company vehicles pick up these pallets (which have different temperature requirements, shelf life and delivery time) from different suppliers/manufacturers. After this, vehicles distribute these pallets at its customer location or distribution centers. But in case, if the truck is not fully loaded than pallets are collected from the supplier's position and brought back to the depots where the same temperature pallets are consolidated. In this research, fully loaded trucks have been used for pickup and delivery purposes.

For the distribution of product orders, the Company's planning team needs to make a scheduling and consolidation plan for pallets' load management along with pickup and delivery. These pallets could be from different suppliers/manufacturers but must have the same delivery location. During the whole planning process, factors that are necessary to be considered are delivery sequence, routing plan, and service time. These factors put a direct impact on the cost and travel time (Zhang & Chen, 2014). This is why the Company is mainly looking to minimize the cost and travel time by the smooth flow of pickup and delivery of the products.

1.2 Problem Statement

As discussed by Dentiz and Ramser (1959), VRP has become a complex problem as it causes to increase the cost in terms of distance, money and time. After the identification of the problem till now, many researches have been done on this issue (G.B.Dantzig & J.H.Ramser., 1959) (M. W. P. Savelsbergh, 1995) (Paolo Toth, 2002) (Bergvinsdottir, Larsen, & Jorgensen, 2004) (GÁBOR NAGY, 2005) (Lars M. Hvattum, 2006) (Chaug-Ing Hsu, 2007) (Lois & Ziliaskopoulos, 2016). But still, this issue is under consideration and has much importance due to cost factors. In real-world applications, by using computerized procedures for planning and distribution processes, companies can save up to 5% to 20% in their transportation cost (Paolo Toth, 2002).

In the current research, the company which is picked as a case-study is facing the cost issue. As discussed earlier, the company is dealing with chilled food products. For which, the company has a wide range of manufacturers/suppliers and distribution centers nodes. The Company's vehicle needs to pick up the product's orders from the source point (supplier nodes) and then deliver it to the distribution centers or the final customer nodes. So, it needs a strong routing/scheduling plan during the pickup and delivery process of orders. In this research, due to limited time, limited data for suppliers (twenty suppliers), distribution centers (twenty distribution centers) and two depots have been used-which means 42 locations nodes have been taken (Dondo & Cerda, 2007) (Prins, 2004). While single order has been considered from the customer nodes.

The company serves the customer nodes by articulated vehicles (each vehicle can move with a load of 26 pallets). These vehicles will travel by considering its full capacity, speed (considered an average fixed speed of 60 mph), service time (considered fixed for each vehicle 30-minutes for one order at pickup or delivery point). Moreover, the company's problem is with pickup and delivery. In this, vehicles pick up the orders from its supplier's nodes and come back to the depot if the order is not delivered on the same day or in case of consolidation of pallets for the fully loaded truck (as considered in the research). This thing may also cause delays and an increase in cost.

Due to the limited framework of the current research, static demand (already known) for orders is being taken for the model. In the whole scenario, the company has to manage multiple scheduling plans because of dealing with multiple depots, suppliers and distribution centers or final customers. In the current scenario, the company's planning team is doing it manually. As discussed earlier, computerized procedures help to save the cost. So, this research is based on the optimization model of vehicle routing/scheduling problems for the company. This research will help to fill the gap of multi depot vehicle routing/scheduling problems with pickup and delivery by using the exact approach. Where the main focus of the problem is to minimize the distance cost by considering the full load truck and designated orders.

1.3 Significance of the Study

1.3.1 Significance for Managers & Practitioners

- This problem has high significance for the managers and practitioners because it will help in cost minimization.
- This research can be useful for scheduling/routing plans of fleets and can increase the efficiency of day to day transportation operations.
- This research will provide a model through which Company can generate systematic scheduling plans for pickup and delivery.
- This model will be applicable for those companies which are facing the multiple depots' vehicle scheduling/routing problem with pickup and delivery.

1.3.2 Significance for Researchers

- This research will contribute to the body of knowledge by combining the different variants of VRP such as Multi-depot VRP, DARP, and VRPPD.
- This research will help researchers to find a solution for multi depot VRP with pickup and delivery with the exact approach.

1.4 Research Purpose

- The research aims to maximum utilize the available vehicles and to reduce the cost in terms of distance traveled by all vehicles.

- Our basic aim is to find not only a smaller decrease but also a quality solution that is capable to solve the problem of VSP specifically in the context of the multi depot, VSP with pickup and delivery and DARP which has been solved in past literature.

1.5 Research Contribution

- This research will help the company to minimize its cost by building a scheduling model for which distances and orders' demand data (generated synthetically) have been taken (Truls Flatberg, 2005). This model can also be applied to companies experiencing the same problem.
- This model will help the Company to make a smooth flow of pickups and deliveries.
- This study will extend the scope of the vehicle scheduling problem by combining the three variants of VRP such as multi depot VRP, VRP with pickup and delivery and DARP. It will contribute towards the body of knowledge.

Chapter No. 2 Literature Review

Logistics problems have a huge importance in practical life. These problems have been studied for many past years. In logistic management, "*an important thing for the company is to provide a smooth and efficient flow of goods*". This is why, companies are working on logistics management and its issues and try to find different methods which can provide the fruitful result for cost reduction and better customer satisfaction (Young HaeLee, 2006).

This chapter is based on the literature review of the Vehicle scheduling/routing problem which is known as a major issue in the field of logistics management. The vehicle routing problem is about "*to find the optimal routes for the given customers by the set of vehicles*" (Paolo Toth, 2002). The vehicle routing problem has much theoretical and practical importance in the field of logistics. VRP is a common problem for management during the distribution of products. It directly impacts the total cost including distributional cost. While the distributional cost includes the maintenance and other routing costs also. This is why, companies need to be very careful because VRP may cause an increase in cost (Bruce Golden, 1984).

2.1 Vehicle Routing Problem

VRP is first, highlighted by the Dantzig and Ramser in 1959 (G.B.Dantzig & J.H.Ramser., 1959). After 1959, Clark and Wright (1964) extended this research in which many models and algorithms were solved to provide an optimal solution for different versions of VRP (Paolo Toth, 2002). Bodin (1974) presented this problem with the taxonomic structure without taking the actual demand. In its next research, Bodin and Golden (1981) presented the routing assignment problem by considering the classification of VRP. After this many researchers worked on it and widely examined this area of research (e.g., Deif and Bodin, 1984; Golden, Baker, Alfaro, and Schaffer, 1985; Laporte, 1987; Min, 1989; Mosheiov, 1994; Anily, 1996; Toth and Vigo, 1999; Osman and Wassan, 2002; J.W.C. van Lint, 2005; Tracares G, 2008; Yiyong Xiao, 2012; Onder Belgina, 2018).

2.2 VRP and its Variants

Vehicle routing problems, its objectives, and the theories are based on the problem analysis, which can be solved with exact or non-exact methods (Reisman, 1992). According to Toth and Vigo (2002), VRP general form is based on "*the design of the routes that help to achieve the minimum routing cost after serving several locations, orders or requests*". While VRP has a wide range of classification which are discussed below:

2.2.1 Asymmetric capacitated VRP

In this type, the cost of links is different from i to j and j to i. The problem has been done through the use of different approaches. Laporte and Nobert (1987) solved this problem thorough the exact algorithm while Vigo (1996) has done this problem by using heuristics. Aragao (2008) has solved the issue by using robust branch algorithms (O., Antonio H. Escobar Z, & E., 2015).

2.2.2 Arcs routing problem

This problem identifies the shortest route by using all the routes and finishes at its starting position. Different real-time problems have been solved on this issue like garbage collection. Toth and Vigo (2002) solved this problem by using the exact approaches. Prins and Ramdane (2001) solved it with a genetic algorithm.

2.2.3 Capacitated VRP (CVRP)

This routing problem covers the elements of the vehicle capacity in which all the customer nodes have definite demand. Baldacci and Mingozzi (2004), Toth and Vigo (2010) have solved this issue through the exact algorithm (O., Antonio H. Escobar Z, & E., 2015). Fukasawa et al (2006) provided the branch cut algorithm approach for the capacitated vehicle routing problem (CVRP) by considering the cutting plain method (C Lee, 2012).

2.2.4 VRP with backhauls (VRPB)

This problem covers the set of customers that needs to be served or the vendors where goods need to be picked up and transported to the distribution center. Wade and Salhi (2003) used the ant algorithm to solve this problem.

2.2.5 VRP with pickup and delivery (VRPPD)

The VRPPD considered a crucial problem in real-life applications, specifically in case of reverse logistics. It is a complex combinatorial optimization problem as compared to other vehicle routing problems (Wassan & Nagy, 2014, Sombuntham & Kachitvichyanukul, 2010). Detailed literature has been discussed on this element of VRP, as the current research covers the area of VRPPD and its varints.

This problem is based on delivery and pickup nodes. In this problem, "*Vehicles take a departure from their warehouses and collect the goods from pickup nodes, after which deliver these goods to the delivery nodes*". According to the researches, pickup and delivery is a complex part of VRP. In which Pickups and deliveries are based on two types of customers: backhauls (suppliers) and line hauls (retailers, distribution centers or the final customers) (Paolo Toth, 2002). Due to the complexity of VRPPD, it has been classified into the following categories:

2.2.5.1 Simultaneously Pickup & Delivery

In this type, the customer has a combined demand for pickups and deliveries which means the vehicle will visit a customer for both pickup and delivery at the same time (Wassan & Nagy, 2014). Min (1989) had solved this problem. This problem based on the real-life application for which one depot, two rigid trucks, and 22 clients have been considered. Min (1989) has solved this issue by making the clusters like travel salesman problem. After this, the issue has been solved again by keeping links length at infinity. After this study, VRP with simultaneously pickup and delivery has been solved by applying different versions of the traveling salesman problem (M. Gendreau, 1999). As we are not considering this type in our study so limited discussion has been made in this area.

2.2.5.2 Mixed pickup and delivery

This approach refers to the situation in which backhauls and line haul customers are on the same route which means pickup and delivery of goods may occur in any order on same the route (Wassan & Nagy, 2014). Further, a load-based insertion procedure has been developed for this type which assists to analyze the load which is still needed to be delivered (B. L. Golden, 1985). After this, the following type of research has been extended through the insertion method presented by Casco, Golden and Wasil (1988) by taking multiple depot VRP with mixed pickup and delivery. Mosheiov (1994) solved this problem by making a minimum spanning tree in which he addressed the overloaded arcs and try to solve this issue by inserting the loaded depot on the arc. Limited literature has been discussed, as the current research does not cover this segment.

2.2.5.3 Pickup First, Delivery second

This type is considered for single demand, in which the vehicle will pick up the goods first from described nodes and then deliver the goods at its delivery nodes. Most researchers assume that it is easy to arrange the customers in the form of backhauls and line hauls on a route from where a vehicle can pick up the products before the deliveries. (Wassan & Nagy, 2014). It also makes the loading/unloading process easy for the vehicles. As compared to, pickups and deliveries, which are done at the same time that may cause overloading and it is difficult to rearrange the products during delivery (GÁBOR NAGY, 2005). Our proposed model in this research will help to extend the past researches because multiple depots have been considered in this research. Company taken for this research is working on the same type of VRP. The company manages the demand load by consolidating in the form of pallets. While full load trucks have been considered for the research which will be delivered at distribution centers or final customers, after the pickup from suppliers or manufacturers.

The vehicle routing problem with pickup and delivery has been solved with different approaches of heuristics in the past, but there is limited research available for exact approaches where VRP has been solved through linear programming. Nagy and Salhi (2003) used various algorithms for the solution of the problem. While Chen and Wu (2006) have solved this problem through hybrid heuristics. Wassan et Al (2014) have solved this issue by using Metaheuristics while integer linear programming formulation has been made. Suprayogi and Andriansyah (2016) have solved the pickup and delivery issue with time windows. In this, mixed-integer linear programming is used for comparison which helps to examine the efficiency of the model. While in recent research, linear programming is directly considered to analyze the VSP with pickup and delivery problems.

2.2.6 Multi-depot VRP

In this type of VRP, it is based on several depots, located in different areas through which customers are going to be served. Normally, each depot assigned by a cluster that contains several customers. Furthermore, the Multi-depot routing problem is known as a modern extension of the vehicle routing problem in which "*different sizes of fleets are considered for the pickup and delivery of product through multiple depots*" (Guedes & Borenstein, 2017). This variant is presented for the first time in the work of Kulkarni and Bhave (1985). According to Kulkarni and Bhave (1985), there must be a binary decision variable for mathematical modeling likeX_{ijk}. After which, Laporte, Nobert and Taillefer (1988) and Carpaneto, Dell'amico, Fischetti and Toth (1989) extended this research.

Renaud, Laporte, and Boctor (1996) have explained this problem by taking G = (V, E) in which V represents nodes and E belongs to the links or the arcs to connect the nodes. According to the author, each link is associated with a cost that can be distance or time traveledC_{ij}. Total number of vehicles K and their capacityP_K. The following constraints have been taken for the research that each vehicle will take start and end to its depot where the customer is going to be served only once. Vehicle capacity must not exceed. Salhi and Sari (1997) solved the issue by using heuristics. Crevier et al. (2005) have solved this issue by taking multiple depots and delivery customers. In its research, vehicles are going to end at their intermediate depots. This problem has been solved through the exact algorithms. According to the author, it is tough to solve multi depot by using exact algorithms. Ombuki Berman (2009) solved it by genetic algorithms.

Baldacci and Mingozzi (2009) presented a mathematical model by using exact methods. In this research, different classes of VRP have been considered with MDVRP. Cornillier, Boctor, and

Renaud (2012) solved the problem by taking heterogeneous vehicles. Further, Contardo, Cordeau and Gendron (2014) solved the dial and ride problem by taking multiple depots and capacitated cases by cut and column generation method. Contardo and Martinelli (2014) have studied the problem with capacitated multi depot VRP by taking length constraints. Li, Li, and Pardalos (2014) presented it by an integer programming model. In this research, time windows and multiple shared depots have been considered. Vidal et al. (2014) provided a new formulation in which it has been solved as a multi depot fleet mix problem. Nieto Isaza et Al. (2015) solved the MDVRP issue by taking heterogeneous vehicles. In this research mixed-integer, linear programming has been used to minimize the routing cost. While in current research, multi-depot VSP also has been considered with pickup and delivery of the products. The literature about the multi-depot vehicle scheduling with pickup and delivery is scant as compared to other variants of vehicle routing problems (GÁBOR NAGY, 2005).

2.2.7 Dial a ride problem (DARP)

DARP is also known as an extension of VRP. "*It contains the element of pickup and delivery requests with a restriction of capacity and mainly used for human services*". This type of VRP is presented for the transportation of ambulatory services, humans, animals, taxis and courier services (Paolo Toth, 2002). One of the common cases for DARP is to take all demand requests known for the customers. These requests need to be served by the same vehicles. On the other hand, Psaraftis (1980) had solved the problem with dynamic programming. In this research, the main objective function was to minimize customer dissatisfaction and route completion time. In this model, customer dissatisfaction is considered as the weight of waiting time along with ride time. During the analysis of the model, the author didn't specify the time windows. While in its later model, it was solved by specifying the time windows (Psaraftis H. N., 1983). Desrosiers et al. (1986) reformulated the single DARP by taking time windows, capacity and precedence constraints. This model was solved through dynamic integer programming. Psarafits (1980) model's issue was not to consider the new requests comes at time *t*. Therefore, it was tough or not possible to re-optimize the model. It causes the problem when the algorithm is slow and requests have a quick chain.

Gendreau et al. (2001) purposed research with dynamic ambulance repositioning that covers various scenarios that are done through parallel computing in an expectation of future requests. Despite having the limitation in Psaraftis (1980), it purposed a dynamic routing problem for later researches. As in most cases of dynamic algorithms, these are solved by the small instances due to its np-hard nature while most of the real-time DARPs' are based on the larger practices (Jean-François Cordeau, 2002). Cordeau and Laporte (2003) presented and solved this problem by using branch and cut algorithms. After this, Jorgense and Larsen (2006) solved it by genetic algorithm.

Guenter Kiechle et al. (2007) has purposed a model by using the constructive heuristics approach. It is based on the move and swap operations for the patient transportation problem related to the Dynamic DARP for emergency cases. It covers two types of services which are based on regular services like pickup and delivery of patients as per routine or predetermined services. While the second service is based on the immediate services in case of emergency. The objective linked with the problems is the minimization of distance cost and response time in case of having an emergency request. This is why it was necessary to arrange vehicle routes in a way that in case of emergency, vehicles can meet the emergency requests within the given time frame. Both objectives are contradictory but not completely. On one hand, vehicles need to complete their task as per routine. On the other side, it is also necessary to keep a margin that can cover the emergency cases, if occur. For a solution, the constructive approach is used. For evaluation purposes, different solutions and strategies are done through simulation. Further, there is a need to continuously update the distance matrix for emergency cases (Guenter Kiechle, 2007). The short discussion has been made on this problem because the problem is solved through heuristics and dynamic programming but covers some material related to DARP.

According to Kristin Berg Bergvinsdottir (2007), DARP is based on the customer request (order) for transportation through a transportation operator. The request is based on the specified pickup (*which belongs to origin location*) and delivery (*belongs to destination*). Again, the problem consists of the best routing plan while keeping the cost low with high-quality services. This problem is based on the customers' door to door transportation where customers can share a ride without having fixed routes. All vehicles may start and end with a different depot. Soft time windows have been used to assess the cost and service time requirements. This problem is solved by the genetic algorithm in which customers' orders and vehicles are encoded and clustering is done through with genetic algorithm by for customer's data. The test is based on 20 random instances between 24 to 144 customers. The results for the DARP cannot be compared for the optimum results as done in the past (Kristin Berg Bergvinsdottir, 2007). This paper covers the element of orders pickup and delivery is done by using a genetic algorithm which is something related to the current research. While for current research, orders need to be picked up from supplier's positions and need to be delivered at the distribution centers or the final customers. Further, for modeling purposes, the LP approach has taken.

2.3 Modeling as a Research Method

To solve the VSP and its extensions, different mathematically models have been made by the researchers. Modeling for research is known for the "*reflection of the effective routing problems by providing an exact mathematical formulation. Where, it is important to understand the various objectives, constraints, and components for the optimization of the problem*". These mathematical formulations are based on algorithms which compete with real-life problems. It is difficult to compare the various models and their techniques because they tend to be made in different standpoints. Models have been developed extensively in real life and social sciences and have considerable success. It is difficult to establish the models in the area of management sciences like logistics, finance or manufacturing. It can be an art or a science. But when using quantitative approach it represents to the science.

In real life modeling, companies have cost issues due to the huge number of requests. It becomes an important issue at the time of optimization. Such kind of problem's modeling is known for the NP-hardness and linked with combinatorial optimization which is difficult to solve (Paolo Toth, 2002). Pop et al. (2012) has used mathematical modeling for generalized VRP. Where two types of models were presented in the paper. The first model made for the node formulation and the second model is made for the flow formulation. Sitek and Wikarek (2017) presented an optimization model for VRP with pickup and delivery. In this model different features of VRP variants have been considered. While in the implementation phase constraint logic programming, mathematical programming, metaheuristics, and databases have been used. Huang and Liu (2015) also used a mathematical model for the VRP with simultaneously pickup and delivery.

There are various components, constraints, and objectives of VRP mathematical formulation that need to be analyzed step by step for optimization.

2.3.1 Objectives: Vehicle routing is based on single or multiple objectives to achieve optimal results. These objectives provide exact routes for the optimization process. The following can be the objective to summarize the vehicle routing problem:

- ***System Objectives:*** Minimization of total routing cost or the operating cost (Toth & Vigo, 2002).
- ***For Customer:*** Minimum service time or the maximum quality of services.
- ***For Environment:*** Reduce congestion and fuel consumption impact on the environment (Toth & Vigo, 2002).

2.3.2 Components: Vehicle routing problem is based on the following components which need to be covered in the mathematical formulation:

- ***Demand model:*** Demand needs to cater from the origin point to destination point and departure time of orders (Victor Pimenta, 2017).
- ***Road Network Model:*** This model is based on the network which assists to fulfill the order requirements. It is based on links and nodes (Michael W Levin, 2016).
- ***Vehicles:*** It provides assistance to fulfill the order demand.
- ***Cost Model:*** Distance travel, traveled time or the linked cost to transport the order form once place to others (Toth & Vigo, 2002).

- ***Optimization Model:*** This component covers the mathematical formulation which helps in the optimization process for the order's routing and helps to achieve the objectives of the model (Toth & Vigo, 2002).

2.3.3 Constraints: The following are the constraints for the vehicle routing problem.

- ***Capacity Constraint****:* All components related to the capacity including customer nodes and vehicles (Michael W Levin, 2016).
- ***Routing model:*** It will provide a road network through which a vehicle is going to pick up the products from the supplier's nodes and deliver it to the distribution centers or the final customers.
- ***Controlling Plan:*** It will assist in vehicle scheduling, routing and time management (Victor Pimenta, 2017).

2.4 Solution Approaches

As vehicle routing problem has been studied in numerous publications. Where most of the studies have a common aim for the VRP is to reduce the routing cost by serving the customers while obeying the different constraints (Toth & Vigo, 2002). All of these past studies are based on different approaches which helped to achieve the common aim for the VRP. Following are the commonly used approaches for the solution of VRP:

2.4.1 Exact approaches

This approach helps to solve complex mathematical problems and provide optimal solutions. For the solution of VSP through the exact approach, a good summary has been presented by the Laporte (1987). Dumas et al. (1991) solved VRP by using the exact approach in which 55 requests have been taken without indicating whether this method can solve the larger requests or not. Various categories for the exact methods discussed below:

2.4.1.1 Direct tree Algorithms

In this type, all the feasible single routes need to be generated based on priority. This type of structure is based on the hierarchy of connected nodes. Each of these nodes belongs to a state. The solution for a search tree is possible through traversing the complete structure from the root node to the child node (CHRISTOFIDES, 1976). Current research is not covered in this category, so it is not discussed in depth.

2.4.1.2 Integer Linear programming

This method is used in a mathematical model for the optimal solution by using the specified constraints. The requirements for the mathematical model are presented in the form of linear relationships (Linear relationship - direct proportional relationship on a graph, which means any change in the independent variable will cause a corresponding change independent variable). Laporte and Nobert (1983) presented this approach. Achuthan and Caccetta (1991), used the integer linear programming for VRP with capacity limit and distance traveled. Later, Fischetti et Al. (1995) have extended it. In this research, the exponential number of sub tour eliminations constraints has been used. Kara and Bektas (2003), used integer linear programming to solve the model of generalized VRP. In recent times, Posada et Al. (2018) have used linear programming for the new extension of VRP known as selective VRP. Further, Anbuudayasankar et Al. (2018) also used this for VRP with simultaneously delivery and pickup for maximum route length. In the current research, integer linear programming has been used to solve the VSP, by considering multi depots and pickup first, delivery second approach.

Exact methods are very much useful for the optimal results of VRP with small size data, but it requires huge computation time to solve the large instances. This is why sometimes these problems give infeasible results due to its np-hard nature. Xu et al. (2003) solved a VRP extension's vehicle routing problem with pickup and delivery by taking heterogeneous vehicles. 500 requests have been considered and solved by using the exact approach. While for results, he used the heuristics approach with column generation (*approach-which is helpful to build the admissible routes*), without giving the guarantee of the optimal solution.

Fukasawa et al. (2004) used the robust branch and cut-price algorithms (*branch-and-cut approach- divide the problem into small problems by fixing the value of a variable*) for capacitated vehicle routing problem (CVRP). Baldacci et al. (2010) used the exact solution approach with a broad class of VRP. Gschwind and Irnich (2015) used branch and cut-price algorithms to solve the dynamic time windows by taking 96 requests. Exact methods are good with results but not very much good with scaling, this is why non-exact methods are commonly used by the researchers with having approximate solutions.

2.4.2 Non-exact approach

It is common, that real-world applications are based on large data and need to be solved on an urgent basis. This is why non-exact approaches are normally preferred with approximates results. If the user runs the heuristics or non-exact methods for a longer period, it would assist with better results. Sometimes non-exact methods are used with some exact methods by taking common assumptions or calculations, like Kilby et Al. (1998) used non-exact approach-truncated branch-and-bound that helps in search by avoiding complete solution trees. Due to the np-hard nature of VRP, most of the work is done through heuristics approaches in the past.

Non-exact or Heuristics approaches are the most commonly used techniques that help to generate and customize the results by using operators. These are based on the set of rules, which are used by describing the choices. Similarly, in contrast, to solve the VRP and its extensions, Metaheuristics (helps in deep research) can be used to get high quality of solutions (Toth & Vigo, The Vehicle Routing Problem, 2002). Kumar and Pannerselvam (2012) have presented the VRPTW and CVRP by framing in the supply chain. They combined the various contributions presented by various authors in three different forms. These forms are heuristics, Metaheuristics, and hybrid. Vidal et al. (2014) used different heuristics in his research which were on 15 variants for VRP. As our research is mainly based on the exact approach so short information has been provided about non-exact approaches.

2.5 Benchmark Instance

As already discussed, researchers have used different approaches to VRP solutions. Similarly, sometimes researchers use real-life data for solution methods, and at times this data is based on benchmark examples. "*Benchmark instances are known as a point of reference which helps to assess the performance".* Christofides (1979), Solomon (1987) and Golden & Assad (1988), all of these researchers have used the benchmark instances for the solution of VRP. Further, Nagata and Braysy (2009) have taken the smaller data set of Chirtofides (1979) and solved it through the memetic algorithms. Li et Al. (2005) generated large scale benchmark data set for VRP by considering more suitable real-life scenarios. OR library (known for the collection of test data sets) generating the benchmark instances for VRP (Diaz, 2006).

Lim et al. (2016) presented benchmark instances for the VRP pickup and delivery. In this study, benchmark examples and manual schedules are compared for non-emergency ambulance services. These benchmark instances provided the results near 0.01% to the optimal solutions. Tarantellis (2005) used tabu search and achieve the results 0.76% which were considered worse. Many other techniques were used with the larger data sets, as compared to the smaller data set results were less competitive but all the results were near 1%. In the current research, benchmark instances have been used with demand data by considering it as a full load truck.

2.6 Summary and Discussion

This chapter is based on the review of related study-vehicle routing/scheduling problems, its variants, its definitions, objectives, components, mathematical modeling, benchmark instances, and various solution methods. In this chapter, a complete picture of the concept of vehicle routing/scheduling problem has been provided. Afterward, we presented different methods detail like exact or the non-exact methods for the solution of VRP through real-life data or benchmarks instances. Exact methods are good with the smaller data and provide optimal results in a reasonable time (Paolo Toth, 2002). This is why non-exact methods are preferred because they can deal with huge data and provide approximate results. As we are looking for the optimal results so exact methods have been discussed more deeply in the current chapter.

Sombuntham (2010) has presented the study in which multiple depots have been considered with simultaneously pickup and delivery. In the research, heterogeneous vehicles have been used for pickup and delivery. The objective functions for this research were minimization of total distance cost, fewer vehicles used and the maximum number of requests served. It was solved by using the heuristics method. Further, benchmark data from Li and Lim (2003) studies have been taken. Lin (2011) has solved VRP with pickup and delivery. In the research, the single depot has been considered with pickup and delivery and time windows constraints. Two types of formulation have been used in this study for the objective function–cost minimization. In the first phase of formulation it is done through the mixed integer programming while in the second phase of formulation, two-stage heuristics have been designed for solutions. Our research is similar to previous studies in terms of its objective functions (Paolo Toth, 2002). In our research, objectives are based on the minimization of total distance cost, less use of vehicles and each order must be served.

The main focus of our research is based on the multi depot vehicle scheduling problem with pickup and delivery. This research has similarities with VRP and VRPDP. But the current focus of research is unexplored, due to considering multiple depots. In this research, the exact approach (Linear Programming) is taken for analysis purposes and optimization of the model. Further, it is based on real-world logistics operations. Where the company is using 44 tonners (based on the capacity of 26 pallets) articulated vehicles/trucks for the pickup and delivery. These vehicles are based on multiple depots from where they perform pickups (supplier's nodes) and delivery (distribution centers nodes) by considering the objective functions. Further, relevant constraints like service time for loading or unloading of vehicle and capacity limits have been considered for this research. In our research, static demand has been taken as a full load truck. The company's actual model is based on dynamic demand but for model simplification, static demand is considered with the help of benchmark instance

Chapter No. 3 Research Methodology & Formulation

The current chapter is based on the research methodologies, problem formulation, and analysis. This chapter has been divided into two phases. The first phase will help you to understand research methodology while the second phase is based on data collection, assumptions, and parameters.

3.1 Phase I: Research Methodology and Model Building

As discussed earlier, the first phase is based on the research methodology and modeling. In the first phase, research methodology has been taken based on research onion layers (Saunders, 2007). While the model building is based on linear programming which is later, solved by using the AIMMS software.

3.1.1 Research Onion Layers

According to Saunders et al. (2007), the research onion helps to understand the research process in more detail. It gives an effective movement that helps you to design a better research methodology. According to Sunder et al (2012), when using research onion layer, user need to go from outer to inner layer of the research onion. The top most layer of the research onion is based on the three dimensions which are known as ontology, epistemology and axiology.

Normally, the research philosophy based on the positivist epistemology in relevant studies. But in current research, it is covering the area of ontological premise. This dimension is known for the reality-based study. It defines the reality which comes to mind and how it impacts the society while doing the research. It differentiates between reality and perception. It helps you to understand the different meanings of an event through which different people are attached. It includes constructivism, pragmatism, and objectivism (Saunders, 2007). This section of philosophy helps to solve the current research problem by considering the pragmatism. Moreover, if there are assumptions for the objective truth in positivist paradigm, it can be recognized through empirical scientific research (Ignatieff, 1998).

According to Saunders et al. (2007), there are various strategies for the researcher to carry out his/ her research. It includes various strategies like experimental research, survey research, and case study-based research, grounded theory, ethnography, and action research. The current research covers the area of case study-based research. The case study-based research strategy is mainly used to focus on one or many peoples or a specific area. It helps to understand any issue in the current working environment. Through the case, study the researcher can establish the significance of culture or context among various issues (Silverman, 2013). This research strategy is good for a comparative study like make a comparison report to check the effect of investment. In the current research, a retail sector company has been taken on which case study research strategy has been applied to solve the vehicle scheduling problem. This strategy will help to compare the current research model with manual scenarios to analyze the performance of vehicle scheduling/routing. Xu and Chen (2001) have used a case study approach for vehicle routing problem with pickup and delivery. Coene and Arnout (2008) also presented a case study-based research on periodic VPR. Nghia et al. (2016), presented a practical case study-based research on VRP with time windows to analyze model performance for distance minimization, total time reduction and less use of vehicles.

One of the layers of research onion is based on research choices. This layer will decide either the research is qualitative or quantitative or mixed (Saunders, 2007). These choices are known as multi-method, mono and mixed. Current research on vehicle routing problems is based on quantitative research as it is based on a mono choice. The vehicle routing/scheduling problem is np hard problem and mainly solved through mathematical formulation (Paolo Toth, 2002). It has been solved in numerous studies where the cost data is based on quantitative choice and taken from real life or considered through benchmark instances (Diaz, 2006) (Lim, 2016) (Paolo Toth, 2002).

The most inner layer of research onion provides the detail for data from where it has been collected, research design and its sample size, its ethics and limitations (Bryman, 2012). This data collection is based on two sources primary or secondary. While the primary source is known for the data which is collected from the source directly. While secondary data is obtained from workplaces or researchers' opinions (Saunders, 2007). In the current research, the secondary source of data has been taken. Further, the demand and order data generated on a random basis (CHRISTOFIDES, 1976) (Solomon, 1987) (Diaz, 2006) (Lim, 2016). In the current research methodology, exact approach-linear programming has been taken for optimal results of the problem's data. As the exact approach takes so much time to provide the solutions. This is why 30-instances generated for sample purposes by considering 42 nodes. According to Golden et al. (1984), 25 sample size is enough to provide good results for algorithms.

3.1.2 Problem Analysis for Model Building

As earlier discussed, the vehicle routing problem and its variants are based on graph theory (Paolo Toth, 2002). Where it is consisting of serving the customer orders within given pickup and delivery, service time and capacity constraints. An order is based on a single customer serving. This means it has a specific pickup and delivery location, order sequencing, service time and capacity constraints. The order picking and delivery is closely related with the VSP with pickup and delivery (VSPDP). It is also common in dial-a-ride problems (Berbeglia & Cordeau, 2007). Where an order/request needs to be fulfilled by the intermediate or different depots for the pickup and delivery. VSP with pickup and delivery may be described as one-to-many problems. In the problem, if an order is linked with a pickup location then it must be served to the defined delivery location (Paolo Toth, 2002). This order can be served from one depot or from different depots where the vehicle needs to end at its started location.

Our real-life problem is based on the working of a UK retail sector company NFT. Where the company needs to pick up the orders from suppliers and deliver it to the distribution centers or the final customers. We model this problem as multi-depot VSP with pickup and delivery by considering static demand. Sombuntham (2010) has solved a similar problem but considered the simultaneously pickup and delivery variant. Where heterogeneous vehicles have been used for the research. The current research problem has so much importance in practical life, where no realistic model has been presented on this issue.

The current chapter is based on the problem’s modeling and optimization. We have decided to model our problem by taking VRP three variants - multiple depots VRP, VRPDP, and DARP. We combined three different variants of VRP for the fulfillment of the company’s requirements, due to having additional constraints. Like vehicle capacity, order fulfillment and service time requirements. The problem is explained in the form of orders which means that each order has a pickup node and delivery node. Each order has the same capacity requirements for all location nodes. It means only one order is going to be served at one time by a vehicle. A key constraint is taken for all nodes that each vehicle will pick the order first, from the pickup node. Afterwhich, this order will be delivered to its designated distribution centers or the final customers.

3.1.3 MDVSPPD Formulation, Equations & Constraints

Although, VSP is known as an NP-hard problem it is possible to formulate the problem through mixed-integer mathematical modeling (linear programming). "*Linear programming is known as an attempt which helps you to give the maximum or minimum solution for the objective function under some constraints. While mixed-integer programming called a subset of linear programming. In mixed integer programming, objective and constraints are linear where some variables may take integer values or some not"* (Paolo Toth, 2002). While the VRP has been presented through the graph theory.

In this section, we proposed sets and indices, parameters, decision, and non-decisional variables and mathematical modeling for multi-depot vehicle routing/scheduling problem with pickup and delivery.

Sets

Depots: set of depots $d \in D$ {1, 2…d}

Vehicles: set of vehicles, $k \in K$ {1, 2, 3…k}

Pickup Nodes: set of suppliers/ Pickup nodes, $p \in P$ {3, 4…p}

Delivery Nodes: set of distribution centres/ delivery nodes, $r \in R$ {23, 24… r}

Orders: set of orders, $o \in O$ {1, 2, 3…o}

For all nodes: set of all locations $N = D \cup P \cup R$ $\forall (i,j) \in N$

For demand nodes: set of all demand locations, $M = P \cup R$

Element Parameters

Parameters which are linked with the sets.

Parking (K): For all vehicles parked at their depots.

Vehicles	Parking Depots
Vehicles parked at depot 1	Depot-1
Vehicles parked at depot 2	Depot-2

POLocations(O): For all orders need to be picked up from suppliers

Order No.	Suppliers/Pickup Nodes
Order 1	Supplier – 1
Order 2	Supplier – 2

DOLocations(O): For all orders need to be delivered at distribution centers

Order No.	Distribution Centers/Delivery Nodes
Order 1	Distribution center- 1

Order 2	Distribution center- 2

Parameters

C_{ij} ,T_{ij}: Distance between nodes i to j & travel time between node i to j.

Q_k: Capacity of vehicles: 26 pallets

$Demand_O$: Full load truck for each order

S_l: Service time for each node

Average speed: 60

Decision Variables:

$\boldsymbol{X_{ij}^K}$ **: Binary Variable -** if vehicle K goes from i to j then = 1, otherwise = 0

$\boldsymbol{Z_O^K}$ **: Binary Variable -** if order O served by vehicle K then = 1, otherwise = 0

Non-decision Variables:

$\boldsymbol{W_{ij}^K}$ **: Non-negative Variable –** Load on vehicle K while traveling from node i to j.

$\boldsymbol{S_{ij}^K}$ **: Non-negative Variable –** Service time for vehicle K while leaving node i to go node j

Objective Function:

Minimization of total distance cost = $\sum_{i,j} \sum_K X_{ij}^K C_{ij}$ i.

The objective function of current research is based on minimizing the distance cost.

Subject to:

$\sum_j X^K_{Parking(K),j} = 1$ ∀ k ∈ K ii.

All vehicles need to depart from their parking depot.

$\sum_K Z^K_O = 1$ ∀ o ∈ O iii.

Each order needs to be served.

$\sum_i W^K_{D,i} = 0$ ∀ k ∈ K, ∀ D ∈ ParkingDepot iv.

Each vehicle departs empty from the depot.

$\sum_j X^K_{Parking(K),j} = \sum_j X^K_{j,Parking(K)}$ ∀ k ∈ K, ∀ D ∈ ParkingDepot v.

Each departed vehicle needs to come back to its parking depot.

$\sum_j X^K_{ji} = \sum_j X^K_{ij}$ ∀ k ∈ K, ∀ i vi.

Each vehicle that will enter a node, same will leave the node.

$W^K_{ij} \le X^K_{ij} * Q_k$ ∀ k ∈ K, ∀ (i,j) ∈ M vii.

The load on the vehicle must not exceed the vehicle capacity.

$\sum_i W^K_{ij} + \sum_O [Plocations(O) = M, \quad Z^K_O * Demand(O)] -$ ∀ k ∈ K, ∀ L viii.

$\sum_O [Rlocations(O) = M, \ Z^K_O * Demand(O)] = \sum_i W^K_{ji}$

Each order needs to be picked up from the suppliers' point and then deliver it to the designated distribution point.

$\sum_i \sum_K \; S_{i,(P)}^{K} \;\leq\; \sum_i \sum_K \; S_{i,(R)}^{K}$ ∀ o ∈ O ix.

Service time of *(P)* for vehicle K must be less than the service time of *(R)* while traveling from i to j. It ensures the pickup first then delivery.

$S_{i,j}^{K} \;\leq X_{ij}^{K} \quad * Q_k * 100$ ∀ k ∈ K, ∀ (i,j) ∈ M x.

To ensure the calculation of service time when the value of X = 1.

$\sum_j S_{i,j}^{K} = \sum_j \; S_{j,i}^{K} + \sum_j X_{ji}^{K} * T_{ji} + S_l$ ∀ k ∈ K, ∀ (i,j) ∈ M xi.

Time balancing, when vehicle K leaves the node to i to go to node j.

$X_{,(P),(R)}^{K} = Z_O^{K}$ ∀ o ∈ O ∀ k ∈ K xii.

Each order needs to be delivered at its designated distribution.

The current model's objective function is based on minimizing the distance cost between depots, suppliers and distribution centers. The constraint (ii) and (iv) ensures that each vehicle departs from their depots and in the end, the same departed vehicles come back to its depot. The constraint (iii), (vi), (viii) and (xii) will confirm that each order must be served when a vehicle enters to a supplier/pickup node. The vehicle must pick up the order from the designated supplier point and then deliver it to the designated distribution centers/delivery node. While it also ensures that each vehicle will enter a node, the same will leave the node.

The constraint (iii) and (vii) are for the vehicle capacity. It ensures that each vehicle when to leave the depot and, in the end, enter the depot must be empty. Moreover, it will not allow to violate the vehicle's capacity while serving the pickup or delivery nodes. The constraint (ix), (x) and (xi) are for time balancing constraint. These constraints will help to calculate the time when a vehicle will leave a node to go to the next node. Further, it ensures that each vehicle will visit pick up node first, then delivery node if X=1.

3.1.4 Why AIMMS?

As already discussed, the vehicle scheduling/routing problem is a combinatorial optimization problem *(“problem which has discrete variables and integers, where the solution sets gave all the possible combinations for discrete variables”)* (Paolo Toth, 2002). It does not allow high-level calculations because it may cause to increase the computational cost but it provides the optimal solutions. This is why the limited size of data has been used for the optimization of this problem and its variants.

In the current research, the selected company is doing vehicle scheduling/routing on a manual basis. It means there are more chances of improvement for vehicle scheduling/routing, which can save the cost for the company. While in current research mixed integer programming (linear programming) is used which referred to the methodology of exact approaches. It will provide the optimal results for vehicle scheduling/routing which may lead to minimizing the cost for the company. The solution of the exact approach (linear programming) software named as AIMMS (Advanced Interactive Multidimensional Modeling System) has been taken. It is known for the perspective analytics software. AIMMS offers two types of products, which help in optimization and modeling. Its analytical perspective assists in the development of the optimization-based application (Roelofs, AIMMS Language References, 2010).

AIMMS helps to design the modeling and provide solutions for optimization and scheduling problems at large scale. Its perspective analytics used algebraic modeling language and create an environment through which a user can create graphical user-interface and can edit the models. It is associated with different solvers like CPLEX, Xpress, and CP Optimizer and many others. It is based on the two types of features mixture: declarative and imperative. The declarative feature helps in the model formulation by using its language elements which can be sets, indices, parameters, variables, and constraints. These are commonly used elements in algebraic modeling (Kallrath, 2004).

AIMMS provides a wide range of support for the optimization of mathematical problems like linear & non-linear programming, mixed-integer & mixed-integer non-linear programming, robust and stochastic optimization, quadratic and constraint programming. Further, AIMMS also allows you to consider the element of uncertainty in mixed and linear programming by using special attributes. This is why AIMMS provides a higher level of solutions along with the customization of solutions. The optimization solution provided by AIMMS can be built in the form of a standalone desktop application or as a software component (Roelofs, AIMMS User's Guide, 2010).

AIMMS perspective analytics platform also helps to use this software in industries which include consumer products, health products, retailers and health care services, oil and chemical, production of steel and agricultural business. Different companies like TNT express used this software for the minimization of carbon footprint and modernized its operation. Further, the Dutch Delta develops a method of calculation to increase the efficiency level of flood protection (AIMMS, 2018). While in current research, it is used for the optimization of a scheduling problem for a distribution company. Where linear programming-mixed integer programming has been used for the solution of the problem.

3.2 Phase II: Data collection, Parameters & Assumptions

The second phase of chapter three explains the data collection and generation, modeling parameters, common assumptions for model and instances to analyze the model performance.

3.2.1 Data Collection & Generation

In the first part of the second phase, data collection and generation procedure has been defined for the current research model. As earlier discussed, our research is based on two depots, twenty suppliers' points, and twenty distribution centers. While twenty orders have been considered in the research. Each supplier and distribution center is linked with the serving of a single order. The current research data is based on the above-described nodes and orders. Further, demand, distance data (drawn by using postcodes for each node), vehicle capacity, speed, service time and travel time have been taken.

The company's nodes data (depots, suppliers and distribution centers) have been taken from the Royal mail (Mail, 2020) and store locator of distribution centers (ASDA, 2019). Royal Mail provides the postcode detail for the nodes. After which, longitude and latitude data for graphical representation has been obtained from the Bing map (BingMap, 2020).

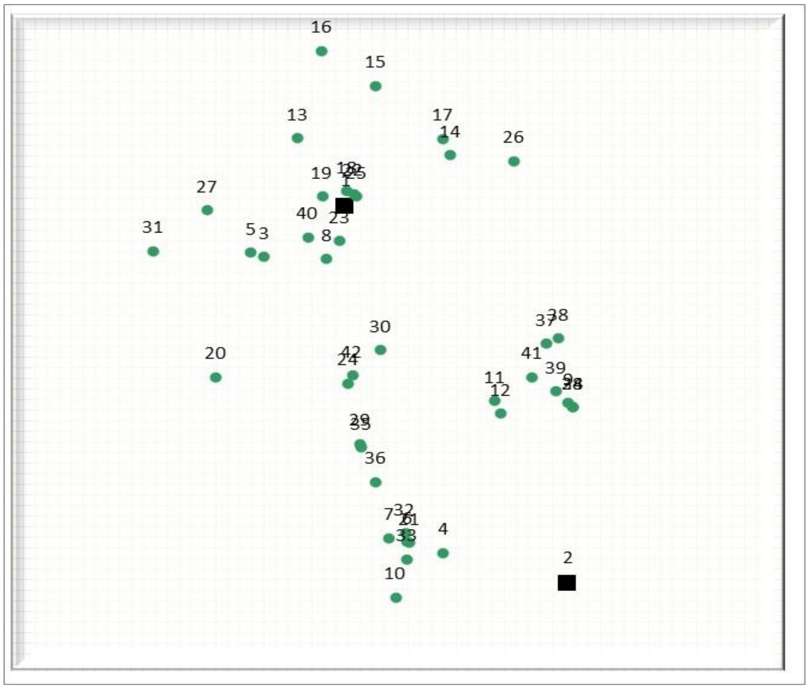

Figure 1: Nodes Data

Additionally, order data for the sample of 30 instances have been generated on a random basis (Mohammad Mirabi, 2016).

P	R	I-1	I-2	I-3	I-4	I-5	I-6	I-7	I-8	I-9	I-10	I-11	I-12	I-13	I-14	I-15	I-16	I-17	I-18	I-19	I-20	I-21	I-22	I-23	I-24	I-25	I-26	I-27	I-28	I-29	I-30
3	23	28	28	23	38	31	36	31	34	42	35	35	34	30	39	34	42	38	29	29	40	28	32	41	23	24	40	33	23	24	39
4	24	26	42	24	39	34	39	28	40	35	41	24	28	38	25	36	26	37	41	30	37	42	26	30	26	42	38	30	37	38	29
5	25	24	41	41	36	23	31	30	28	37	25	36	41	41	29	32	37	32	36	27	25	40	42	39	39	27	28	35	34	42	42
6	26	35	27	38	31	35	38	23	24	38	23	38	33	39	26	25	36	29	37	31	36	30	23	26	24	26	39	37	29	35	23
7	27	33	34	39	35	24	28	34	23	40	37	29	42	29	34	27	41	39	38	25	39	35	29	31	25	32	31	41	24	23	36
8	28	37	30	29	32	32	30	38	41	30	34	39	36	40	36	37	38	42	27	34	42	32	33	24	28	36	25	25	25	33	27
9	29	42	24	34	28	36	32	27	29	25	33	40	27	35	32	33	31	33	24	28	28	34	36	27	34	25	37	28	39	26	34
10	30	31	32	37	33	39	42	42	38	26	24	32	35	28	42	30	35	23	32	41	32	25	37	35	38	41	35	31	35	39	32
11	31	32	38	35	26	37	29	39	31	27	27	25	30	34	23	24	27	40	33	42	38	26	30	34	37	40	29	29	30	29	41
12	32	27	39	25	30	28	26	36	26	39	36	26	31	42	28	31	28	28	28	24	31	31	39	33	42	30	26	40	36	36	31
13	33	34	25	32	23	27	23	29	27	32	28	23	23	37	41	38	24	35	30	33	33	24	38	40	31	33	27	34	38	28	24
14	34	39	26	27	37	41	41	33	35	33	40	28	26	36	24	35	30	24	31	40	24	23	31	32	30	35	41	39	33	37	28
15	35	36	33	36	40	25	37	32	42	28	31	41	32	23	38	26	39	25	40	35	27	36	40	23	40	38	34	27	42	30	33
16	36	38	23	30	41	30	40	41	37	29	39	42	25	33	35	23	40	31	23	38	34	33	34	25	29	29	24	36	40	41	37
17	37	29	40	26	27	26	34	37	33	34	26	33	37	24	40	42	33	41	35	23	26	38	35	36	32	23	23	24	28	27	25
18	38	23	36	42	25	33	25	35	39	36	30	37	24	31	30	29	34	34	42	37	29	41	24	38	41	39	36	42	26	40	38
19	39	25	35	28	24	40	27	25	30	31	32	34	40	32	33	41	29	27	39	39	30	37	27	37	35	34	32	23	31	31	40
20	40	41	37	40	29	38	35	40	36	41	38	31	29	27	31	39	32	30	25	32	41	27	28	28	27	37	30	32	27	25	35
21	41	40	29	33	34	29	24	26	32	24	29	27	38	25	27	40	25	26	26	26	35	39	25	42	36	31	33	26	32	32	26
22	42	30	31	31	42	42	33	24	25	23	42	30	39	26	37	28	23	36	34	36	23	29	41	29	33	28	42	38	41	34	30

Table 1: Order Instances

In **Table 1**, a randomly generated order set is given which has been taken for 20 orders. In the table, Column P is showing suppliers nodes that are kept constant for each instance. While column R shows the distribution nodes whose values vary in each instance. As for the first order in instance 1, the supplier node 3will serve distribution node 28. While in the third instance, supplier node 3 will serve distribution center node 23. Further, demand for all orders has been considered a full load truck. According to Li and Lu (2014), the full truckload may lead vehicle routing problem towards cost minimization.

In the current research, vehicle capacity for each order is considered as 26 pallets which are common in frozen food transportation (James F. Thompson, 2002). While all vehicles will travel at a fixed average speed of 60 mph. The speed limit has been set based on traffic rules at motorway and by taking the example of benchmark instances (Yanling Feng, 2017) (Toth & Vigo, 2002). In the current model, service time for all vehicles is assumed to be constant for each order loading and unloading (Toth & Vigo, 2002). According to Errico et al. (2016), actual service time may not lead to a viable solution. While travel time has been calculated based on service time and distance between two nodes (Toth & Vigo, 2002).

3.2.2 Assumptions

The next part of phase two is based on the general assumptions taken to solve the current VRP model. These assumptions help to provide the optimal schedules that are generated through model outputs (Paolo Toth, 2002). Following are the assumptions:

- All vehicles are homogenous. All Vehicles’ capacity is the same and fixed.
- All vehicle speed is considered the same and static.
- All demand locations and depots are predefined.

- All vehicles' demand considered a fully loaded truck.
- All vehicles' have fixed service time for each order loading and unloading.
- Transportation costs for the vehicles are based on distance travelled.

3.2.3 Parameters

The third part of phase two is based on the parameters which are associated with the current problem model. The parameters can help to affect the behaviour or the performance of the model. These parameters are demand, capacity, distance, service time, travel time. Hence, it is probably not possible to have all the real-time information about the parameters for a company. This is why few parameters have been generated on a random basis (Toth & Vigo, The Vehicle Routing Problem, 2002). As earlier discussed, the following are the key parameters for the current model:

$\boldsymbol{C_{ij}}$,$\boldsymbol{T_{ij}}$: Distance between nodes i to j & travel time between node i to j.

As earlier discussed, distances have been drawn through postcodes for the company, which have been taken from Royal Mail and store locator for distribution centers (ASDA, 2019) (Mail, 2020). After this, postcodes used to design the distance matrix which has been recovered from the Bing map (BingMap, 2020).

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42
1	0	220	40	179	43	165	158	32	164	193	164	157	48	42	73	83	47	8	18	105	165	8	23	99	7	65	64	168	120	71	76	161	172	168	122	138	145	141	157	28	161	101
2	221	0	181	49	187	62	72	185	75	78	84	78	245	194	229	278	200	222	216	152	62	223	193	128	222	177	208	77	100	131	231	63	69	77	100	83	99	99	82	195	86	130
3	41	182	0	141	13	127	120	29	142	156	117	119	71	76	90	104	81	42	35	67	127	42	30	62	41	99	34	145	82	68	46	123	135	145	85	100	147	151	146	22	150	63
4	180	50	140	0	146	21	31	144	76	33	70	71	204	194	230	237	200	181	175	111	20	182	152	87	181	178	167	79	59	107	190	22	19	79	58	45	100	100	83	154	86	89
5	43	188	14	147	0	133	126	35	148	161	123	125	64	78	93	98	83	44	37	73	133	44	33	67	43	101	27	151	88	74	37	129	140	151	90	106	153	150	152	24	155	69
6	166	62	126	21	132	0	17	130	79	51	73	75	190	198	216	223	204	167	161	97	1	168	138	73	167	181	153	82	45	93	176	4	8	82	45	31	103	103	86	140	89	75
7	160	73	119	32	125	18	0	123	90	38	79	81	183	195	209	217	200	161	154	91	18	161	132	66	160	192	146	93	39	87	169	7	14	93	38	24	114	114	97	134	100	69
8	31	185	29	144	34	130	123	0	145	159	120	122	63	73	88	98	79	39	33	78	130	40	10	65	39	96	55	148	85	47	79	126	138	148	88	104	120	117	133	13	137	66
9	165	76	142	77	148	80	90	146	0	103	29	28	181	128	164	187	134	166	151	132	80	160	132	81	159	112	169	7	80	70	192	81	87	7	82	85	33	33	9	157	20	85
10	195	81	155	32	160	50	38	158	103	0	97	98	218	221	245	252	227	196	189	126	50	196	167	101	195	205	181	106	74	122	204	43	21	106	73	59	126	127	110	169	113	104
11	163	85	117	69	123	66	79	121	28	96	0	8	181	126	162	185	132	165	152	107	66	159	130	56	157	110	144	32	55	44	167	67	73	32	57	58	31	35	30	132	18	60
12	158	78	117	69	123	66	80	121	28	96	7	0	181	130	166	189	136	159	152	107	66	163	130	56	162	114	144	32	55	47	167	67	73	32	57	58	35	39	29	132	32	60
13	48	245	66	204	65	191	183	68	180	219	180	182	0	84	38	40	89	36	34	130	190	50	60	125	49	106	57	184	146	108	89	186	198	184	148	164	160	157	173	53	176	126
14	41	195	75	196	78	199	193	69	128	222	127	131	83	0	40	64	11	43	53	140	199	37	61	134	36	28	99	132	155	109	111	196	206	132	157	173	108	105	121	63	124	136
15	73	230	90	229	93	215	208	93	163	244	162	166	41	40	0	26	32	60	58	155	215	74	84	149	73	63	91	167	170	132	123	211	222	167	172	188	143	139	155	78	159	151
16	83	253	104	237	98	224	216	103	186	252	186	189	40	64	27	0	55	70	69	163	223	85	94	158	83	87	90	190	179	142	122	219	231	190	181	197	166	163	179	88	183	160
17	47	199	81	200	83	203	198	75	132	227	132	135	89	10	32	56	0	48	58	145	203	42	66	140	41	33	104	136	160	114	116	204	210	136	163	178	113	109	125	68	129	141
18	8	222	42	181	45	167	160	36	166	195	165	158	36	43	61	71	48	0	8	107	167	3	28	101	8	66	66	170	122	76	78	162	174	170	124	140	146	142	158	30	162	103
19	18	215	36	174	39	161	153	38	150	189	150	152	34	53	59	68	59	8	0	100	160	20	30	95	19	76	60	154	116	78	71	156	168	154	118	134	130	127	143	23	146	97
20	105	153	65	112	71	98	91	77	134	126	109	111	129	141	155	162	146	106	100	0	98	107	86	54	106	163	92	137	62	67	60	94	105	137	64	71	139	143	139	87	142	56
21	167	63	127	22	133	2	18	131	80	51	74	75	191	198	217	224	204	168	162	98	0	169	139	73	167	182	154	83	46	94	177	5	7	83	45	31	103	104	87	141	90	76
22	10	223	44	182	46	168	161	37	160	197	160	160	51	38	76	86	43	4	21	108	168	0	29	102	2	61	67	164	123	77	79	164	176	164	126	141	141	137	153	31	157	104
23	22	194	30	153	33	139	132	11	131	167	129	131	58	73	83	93	78	39	28	87	139	39	0	73	38	95	54	135	94	50	65	135	146	135	96	112	111	108	124	18	127	75
24	101	128	61	87	67	73	66	65	80	101	55	57	125	137	151	158	142	102	96	57	73	103	73	0	102	159	88	83	28	23	111	69	80	83	30	46	85	89	85	75	88	4
25	8	222	42	181	45	167	160	36	159	195	159	159	50	37	75	85	42	10	20	107	167	2	28	101	0	60	66	163	122	76	78	163	174	163	124	140	139	136	152	30	156	103
26	65	178	99	179	102	182	193	93	111	205	111	114	107	28	64	87	34	67	77	164	182	61	84	158	60	0	123	115	164	112	135	183	189	115	166	167	91	88	104	86	107	160
27	64	209	35	168	29	154	147	56	169	182	144	146	57	100	88	90	105	65	59	94	154	66	54	88	65	122	0	172	109	95	39	150	161	172	111	127	174	178	173	46	176	90
28	168	78	144	78	150	82	92	148	7	105	33	32	185	132	167	191	138	170	154	134	81	164	136	83	163	115	171	0	82	73	194	83	89	0	84	87	37	37	13	159	23	87
29	122	100	82	59	88	46	38	86	79	74	54	56	146	157	172	179	162	123	117	62	45	124	94	28	122	180	109	82	0	49	132	41	53	82	3	19	84	88	84	96	87	31
30	71	132	68	107	74	93	86	45	69	122	43	48	107	117	132	142	123	83	77	71	93	84	50	19	83	112	95	73	40	0	118	89	101	73	42	66	74	78	70	57	62	17
31	76	231	46	190	37	176	169	79	191	205	166	168	82	111	113	115	116	77	70	60	176	77	66	111	76	134	31	194	131	117	0	172	184	194	134	150	196	200	195	57	199	112
32	162	66	121	25	127	6	13	125	83	48	61	63	185	197	211	219	202	163	156	93	7	163	134	68	162	185	148	86	41	88	171	0	14	86	40	26	107	107	90	136	93	71
33	173	69	133	31	139	8	13	137	86	21	80	81	197	204	223	230	210	174	168	104	7	175	145	79	173	188	160	89	52	100	183	11	0	89	51	37	109	110	93	147	96	82
34	168	78	144	78	150	82	92	148	7	105	33	32	185	132	167	191	138	170	154	134	81	164	136	83	163	115	171	0	82	73	194	83	89	0	84	87	37	37	13	159	23	87
35	124	100	84	59	90	45	38	88	81	73	57	58	148	160	174	181	165	125	119	64	45	126	96	31	125	165	111	84	2	51	134	41	52	84	0	15	86	91	86	98	89	33
36	140	84	100	45	106	31	24	104	84	59	58	60	164	176	190	197	181	141	135	72	31	142	112	47	141	167	127	87	19	67	150	27	38	87	15	0	88	92	87	114	90	49
37	145	99	147	100	153	103	113	121	32	126	31	35	161	108	143	167	114	146	130	136	102	140	112	86	139	91	174	36	85	74	197	104	110	36	87	88	0	5	25	132	28	90
38	139	103	151	104	157	107	117	115	36	130	35	39	155	102	138	161	108	140	125	140	107	134	106	90	133	85	178	40	89	78	201	108	114	40	91	92	5	0	29	126	32	94
39	157	82	147	83	153	86	97	133	8	110	30	29	173	120	155	179	126	158	143	136	86	152	124	85	151	103	174	12	84	70	197	87	93	12	86	91	25	25	0	144	12	90
40	28	196	21	155	24	141	134	14	156	170	131	133	53	64	78	88	69	29	23	85	141	30	18	75	29	86	45	159	96	57	56	137	149	159	99	114	132	128	144	0	148	77
41	160	86	151	87	156	90	101	136	19	113	19	32	176	123	159	182	129	162	146	140	90	156	128	89	155	107	178	23	88	62	200	91	97	23	90	95	28	28	12	147	0	93
42	104	131	64	89	70	76	68	68	84	104	59	61	128	139	154	161	144	105	99	60	76	106	76	4	104	162	91	87	31	17	114	71	83	87	33	49	89	94	89	78	92	0

Table 2: Distance Matrix

In **Error! Reference source not found.**, node (1, 2) represents the depots, (3-22) donated the suppliers' nodes and (23-42) have been taken for distribution centers. Further, the distance matrix allows calculating the travel time.

Travel time has been calculated by using the following formula:

Total time = Travel Time (i.e. distance/Average Speed) + service time

$\boldsymbol{Q_k}$: Capacity of vehicles: 26 pallets / Full load Truck

As earlier discussed, in the current problem we consider the full load truck for which vehicle capacity has been considered 26 pallets (James F. Thompson, 2002).

$\boldsymbol{Demand_O}$: Full load truck for each order

Our research is based on static demand to keep the model simple. This is why, full load truck has been taken (Toth & Vigo, 2002). The vehicle will pick up or drop the demand order as articulated vehicles have been used by the company.

$\boldsymbol{S_l}$: Service time for each node is fixed

For each order either pickup or delivery, service time kept at constant. It is tough to take the actual data for the service time at each node while using the exact approach (Errico, Desaulniers, & Gendreau, 2016).

Speed: Considered fixed speed

In the current model, road speed has been taken on an average basis and considered fixed for each vehicle. According to traffic laws, at the motorway, the heavy vehicle may travel with a maximum speed limit of 60 mph. While in our model our common assumption is that we are using motorway for vehicle routing. This is why 60 mph speed has been taken for the model (Yanling Feng, 2017).

3.2.4 Scenarios

The last part of phase two is based on the instances that will help to analyze the model's performance. The current problem’s model will be compared with these instances to test the model's viability. Following are the instances which are going to be compared:

- Suppliers to Distribution Centers
- Two Orders for each Vehicle
- AIMMS Model – Current Research Problem

3.2.4.1 Supplier to Distribution Centers

In this scenario, each vehicle will travel from suppliers to distribution centers. There are no depots. It means that every order is picked up directly from the suppliers' point and delivered to the distribution centers. After which the vehicle will return to the supplying point from where it started. In this instance, the common assumption is that customer nodes/demand locations are managing their orders by insourced vehicles.

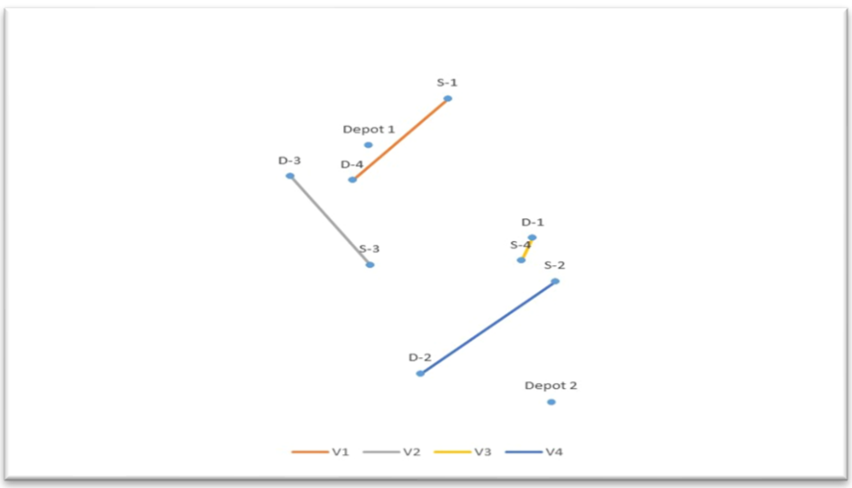

Figure 2: Supplier to Distribution Centers

3.2.4.2 Two Orders for Each Vehicle

This scenario is based on the assumption that each vehicle will serve two orders for the company. For this, each vehicle will take depart from their depot and pick up the orders from the suppliers' point and deliver it to their distribution centers. After this, the vehicle will travel again towards the suppliers' point for the next order. After the order is delivered, it will return to its assigned depot.

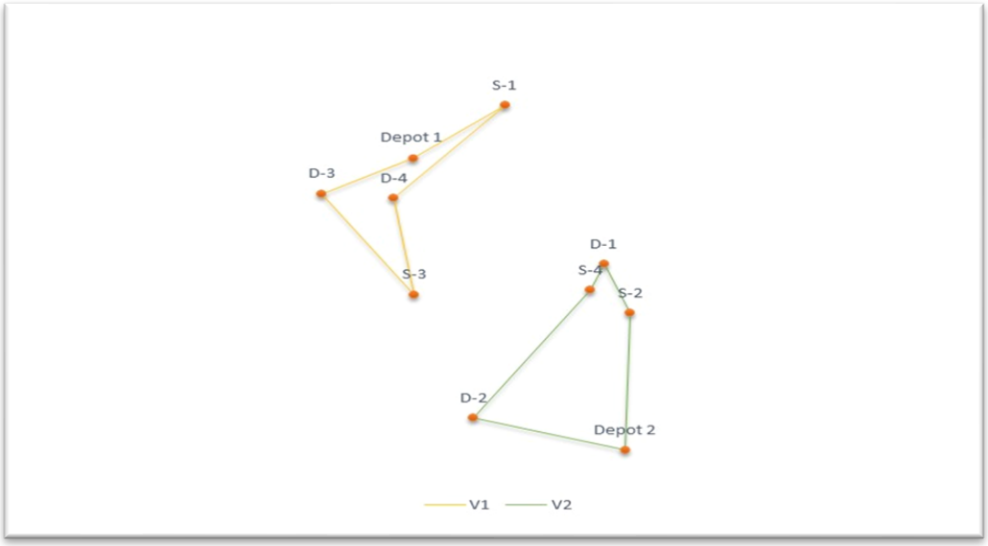

Figure 3: Two Orders

3.2.4.3 AIMMS Model

The final instance is based on the original model designed to solve the current problem of the company. As discussed earlier, each vehicle in the model will start from the depot and end at the same depot. While a vehicle can serve more than one order to fulfill the objective function.

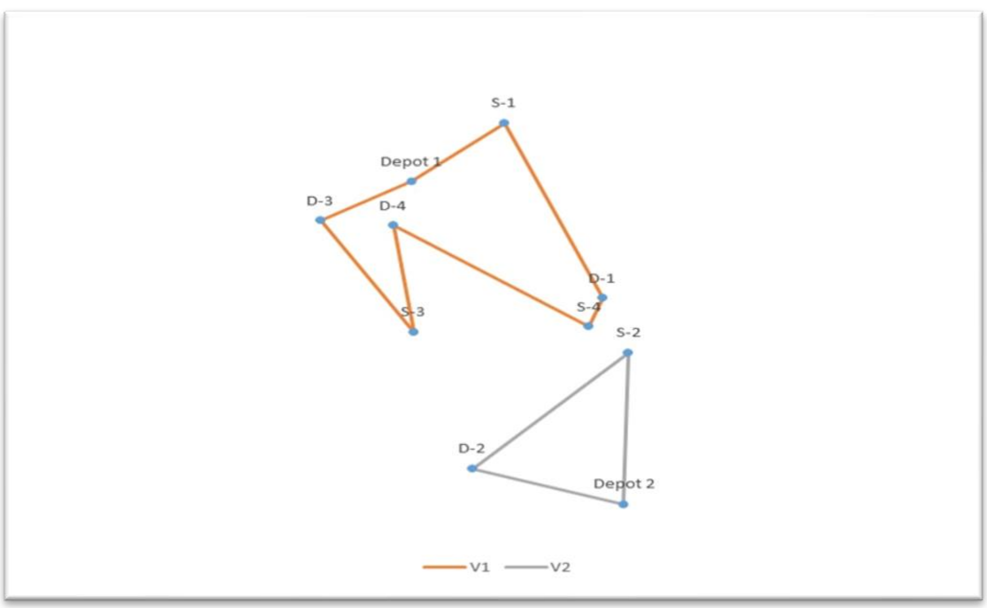

Figure 4: AIMMS Model

The two scenarios and current research problem’s model are intended to minimize the distance cost for the company. While in each instance, vehicles have the same order demand, capacity and speed requirements. In addition, 30 random order examples are sampled for each instance, in which each order assigned to its suppliers' point and distribution centers. The sample data will help to test the model's performance by analyzing it with other instances. On the other hand, talking about the differences in these instances is that, there is no depot in the first instance, while in the second example, each vehicle may serve only two orders.

Chapter No. 4 Results & Analysis

Chapter 4 is based on the results and its analysis. In this section, all the instances' results have been compared with each other. This section will help to evaluate the performance and operational capability of the linear programming-based model run through AIMMS for optimal results. The given figure will help to understand the following model working:

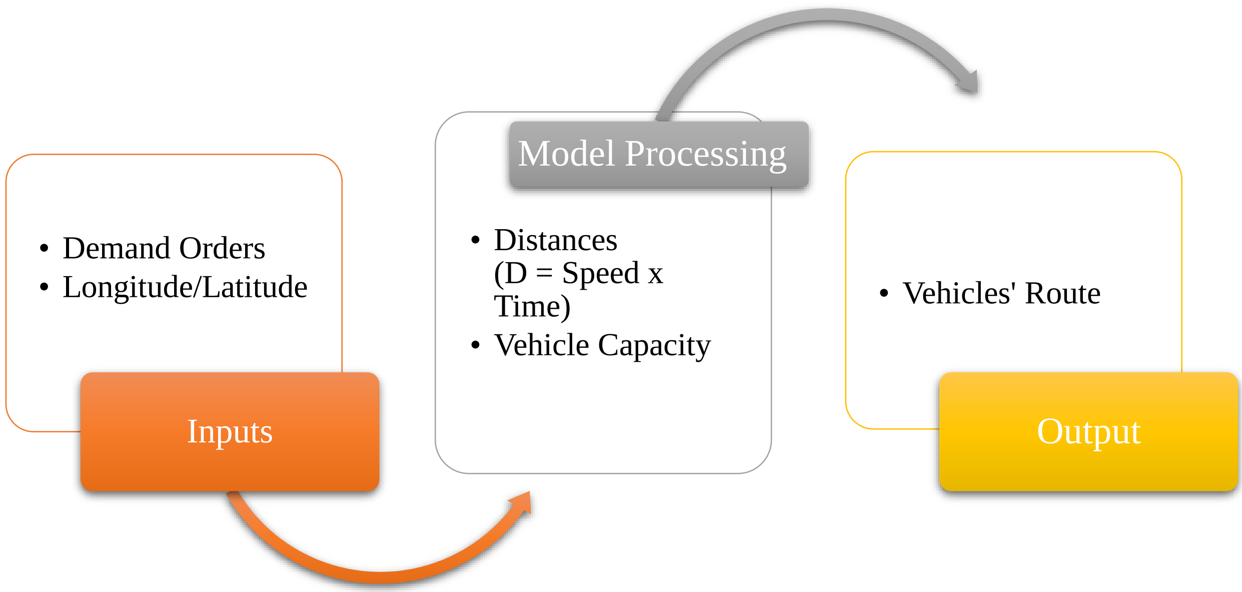

Figure 5: Model Processing

The two scenarios which have been taken to analyze the viability of the current model are solved by manual procedures. The manual procedure has done by calculating the distances between each node. From which closet distance nodes have been taken to serve the orders. Further, the distance traveled between each node is considered as the distance-per kilometer between each node. While data for all nodes have already been rendered. The graphical representation is shown for order-based instances to highlight the effectiveness of the proposed model.

The first order-based instance is based on the following values:

P	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
R	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42
I-1	28	26	24	35	33	37	42	31	32	27	34	39	36	38	29	23	25	41	40	30

Table 3: Order Instance 1

4.1 Scenario 1

As earlier discussed, P values representing the suppliers and R values are for distribution centers which are varying. In this table P 3 will serve R 28, P 4 will serve R 26 and so on. For the first instance distance results is given below which is known for suppliers to distribution centers for direct pickup and delivery:

P	R	I-1	1-sided	2-sided
3	23	28	145	290
4	24	26	178	356
5	25	24	67	134
6	26	35	45	89
7	27	33	14	28
8	28	37	120	241
9	29	42	85	170
10	30	31	204	409
11	31	32	67	134
12	32	27	144	289
13	33	34	184	368
14	34	39	121	241
15	35	36	188	377
16	36	38	163	326
17	37	29	160	321
18	38	23	28	55
19	39	25	19	37
20	40	41	142	284
21	41	40	141	282
22	42	30	77	154
Total Distance Cost			2,293	4,585

Table 4: P to R

Error! Reference source not found. shows the instance results for the orders. This table has one-sided and two-sided spacing. The distance on one side represents the route from the suppliers to the distribution centers, while the two-way distances indicate that the vehicle started from the supplier nodes, places the order at the delivery node and returns from where it started. In addition, the total distance traveled for one-way routes is 2293 and for the full round, it was 4585. For this example, a common assumption is that each supplier will serve its designated distribution centers through the vehicle they maintained. It means that overall twenty vehicles have been used to run this instance for cost minimization.

4.2 Scenario 2

In the next scenario which has been taken with the instance of the same order 1. In this instance, each vehicle will serve only two orders. As earlier discussed in this instance vehicle will depart from the depot pick up the order from suppliers then deliver the order at distribution centers. After which vehicle may travel towards its next order with the same guidelines for vehicle capacity, speed and demand. When the vehicle will serve both orders then it will turn back to the depot from where it started. The results for the second scenario are given below:

Orders	P	R	I-1	Vehicle#	Distances
1	3	23	28	V-1	466.8
20	22	42	30	V-1	
9	11	31	32	V-10	499.01
10	12	32	27	V-10	
3	5	25	24	V-2	551.03
11	13	33	34	V-2	
12	14	34	39	V-3	571.05
15	17	37	29	V-3	
13	15	35	36	V-4	711.87
14	16	36	38	V-4	
16	18	38	23	V-5	90.64
17	19	39	25	V-5	
5	7	27	33	V-6	534.58
8	10	30	31	V-6	
4	6	26	35	V-7	488.71
19	21	41	40	V-7	
6	8	28	37	V-8	649.86
18	20	40	41	V-8	
2	4	24	26	V-9	553.92
7	9	29	42	V-9	
Total Distance					**5117.47**

Table 5: Two Orders

As discussed earlier, each vehicle will place two orders in this example. This means that a total of 10 vehicles were used to fulfill the orders. In **Error! Reference source not found.** distances are calculated by sending the vehicle from the depot to its nearest supplier location and distribution centers. After delivering your orders, the vehicle will return to its depot. In the current example, the total distance cost is 5117.47, which is much higher than the previous scenario. The overall distance cost for both events shows that scenario 1 is providing better results than scenario 2. The picture below shows the graphical representation for scenario 2.

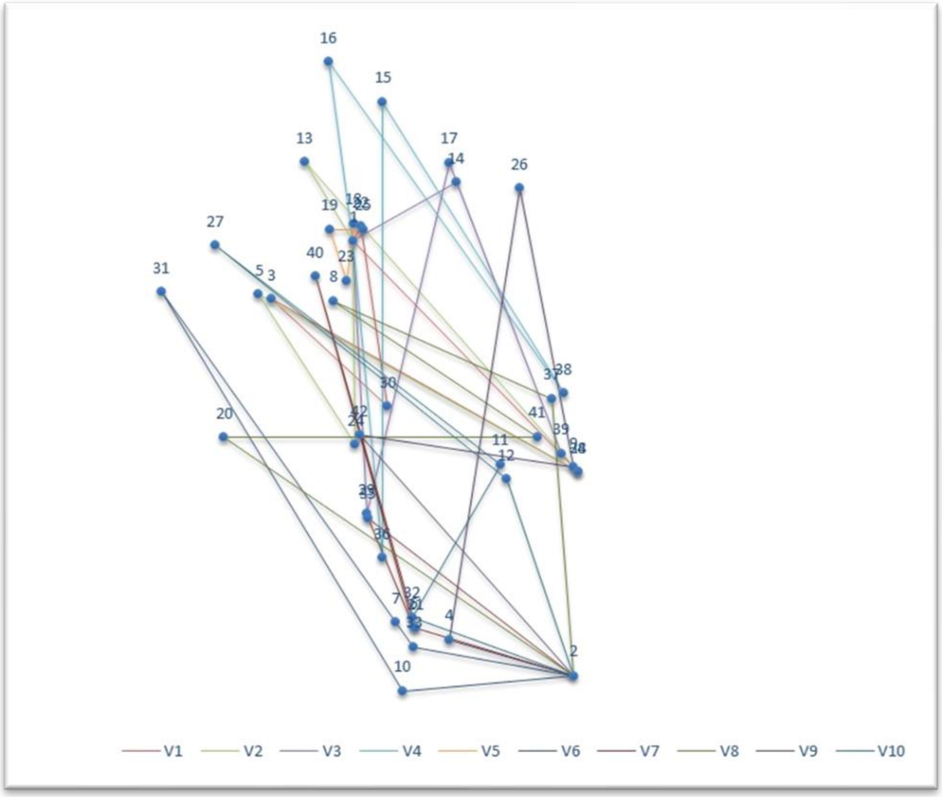

Figure 6: Two Orders

4.3 AIMMS Model

After scenario 2, the last scenario has been described. This instance has been designed through linear programming to solve the current issue of distance minimization for the company. As previously discussed, in this model a vehicle may serve more than one order without exceeding the capacity limit, speed, and service time and order demand requirements. The same order instance 1 has been taken to analyze the current research problem. The results for the current model have been shown below:

Orders	P	R	I-1	Vehicle#	Distances
6	8	28	37	V-1	1662.45
9	11	31	32	V-1	
10	12	32	27	V-1	
12	14	34	39	V-1	
14	16	36	38	V-1	
15	17	37	29	V-1	
17	19	39	25	V-1	
18	20	40	41	V-1	
19	21	41	40	V-1	
20	22	42	30	V-1	
16	18	38	23	V-2	57.3
1	3	23	28	V-3	1610.68
2	4	24	26	V-3	
3	5	25	24	V-3	
4	6	26	35	V-3	
5	7	27	33	V-3	
7	9	29	42	V-3	
8	10	30	31	V-3	
11	13	33	34	V-3	
13	15	35	36	V-3	
Total Distance Cost					**3330.43**

Table 6: AIMMS Model

The current model, built on the basis of linear programming and run through AIMMS software, provides an overall distance cost of 3330.43 for complete routing. Compared to other instances, this model reduced the cost of distance compared to other models. In addition, fewer vehicles were used to run the current model of MDVSPPD. Below given map show the current model routing.

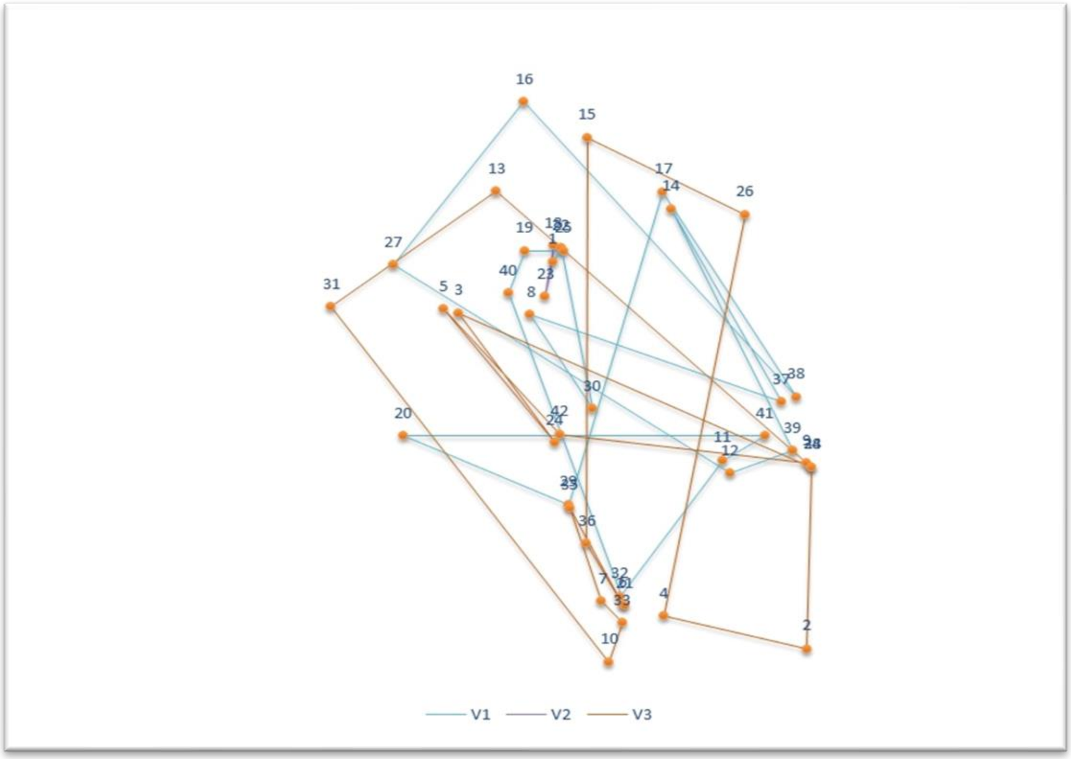

Figure 7: AIMMS Model

4.4 Distances Results for all Instances

Further for the next instances of orders' data set and sample instances, the table has been presented in which total distance cost data has been shared for each instance. For all scenarios, the same vehicles have been used.

Instances	Distances		
	Scenario 1	Scenario 2	AIMMS Model
1	4586	5117	3329
2	3708	4372	2902
3	3976	4560	3023
4	3353	4096	2807
5	3144	3777	2613
6	3495	4010	2800
7	4558	5115	3317
8	4670	5386	3371
9	4849	5752	3469
10	4223	4766	3147
11	4667	5522	3373
12	3903	4555	3008
13	4165	4792	3119
14	4870	5452	3480
15	5165	5746	3544
16	4692	5381	3387
17	4463	4881	3268
18	4051	4484	3063
19	4610	5377	3341
20	3346	4159	2709
21	4606	5186	3339
22	4579	5203	3330
23	4752	5364	3409
24	4409	5218	3242
25	4904	5862	3490
26	3774	4745	2924
27	4497	5066	3291
28	3490	4268	2780
29	3933	4703	3013
30	3806	4524	2955

Table 7: Instances Results

The responses obtained using all the orders data set are mentioned above for each example. From all scenarios, all the 30 instances of linear programming-based models yield the maximum results that show the robustness of the model. Scenario 1 which was based on serving the order from suppliers to distribution centers is better than serving the two orders by each vehicle scenario.

While the third model is better designed for the company to minimize its distance cost from all the manual scenarios.

4.5 Distance Indexing Results

To illustrate the reliability of the model, the indexing method has been used for all instances. After which the average is taken for each scenario.

Instances	Distances			Distances' Indexing		
	Scenario 1	Scenario 2	AIMMS Model	Scenario 1	Scenario 2	AIMMS
1	4586	5117	3329	100	112	73
2	3708	4372	2902	100	118	78
3	3976	4560	3023	100	115	76
4	3353	4096	2807	100	122	84
5	3144	3777	2613	100	120	83
6	3495	4010	2800	100	115	80
7	4558	5115	3317	100	112	73
8	4670	5386	3371	100	115	72
9	4849	5752	3469	100	119	72
10	4223	4766	3147	100	113	75
11	4667	5522	3373	100	118	72
12	3903	4555	3008	100	117	77
13	4165	4792	3119	100	115	75
14	4870	5452	3480	100	112	71
15	5165	5746	3544	100	111	69
16	4692	5381	3387	100	115	72
17	4463	4881	3268	100	109	73
18	4051	4484	3063	100	111	76
19	4610	5377	3341	100	117	72
20	3346	4159	2709	100	124	81
21	4606	5186	3339	100	113	72
22	4579	5203	3330	100	114	73
23	4752	5364	3409	100	113	72
24	4409	5218	3242	100	118	74
25	4904	5862	3490	100	120	71
26	3774	4745	2924	100	126	77
27	4497	5066	3291	100	113	73
28	3490	4268	2780	100	122	80
29	3933	4703	3013	100	120	77
30	3806	4524	2955	100	119	78

Table 8: Indexing Results

As we can see, in **Error! Reference source not found.**, the results of all three scenarios are shown by indexing. The index provides an efficient look to assess all instances' results. It has been used to statistically measure the change in distance cost in all scenarios. Indexing is calculated using the formula:

Distance Index formula = distance cost for the current situation/distance cost for the base scenario.

For all scenarios, scenario one (suppliers to distribution centers) is considered a base year. The table values represent that there are no fluctuations in scenario 1 results. It means there is no rise or fall in all the 30 instances. In the second scenario, the price of the index varies for each instance. All the trends in Scenario 2 have a common attitude that each instance has a higher value. In addition, the index value is different for all instances for scenario 3 (AIMMS model). The common thing in this scenario is that it results in a lower price index for each instance then the base value. The table below gives the average of each scenario for all instances.

Indexing Averages		
Scenario 1	**Scenario 2**	**AIMMS**
100	116	75

Table 9: Indexing Averages

In **Error! Reference source not found.**, the average index results for all scenarios have been given. Based on these averages, the results of Scenario 1 show that it costs 16 times more distance then scenario 2. However, the distance cost of scenario 1 varies greatly when compared to the current problem model-AIMMS. In addition, Scenario 2 results show that they have a lower distance cost than scenario 1, but still cost more distance than the AIMMS model. The AIMMS model costs 25% less distance than scenario 2. Based on the average results of AIMMS, it is the most reliable model for the company to reduce its distance cost because in each instance it offers a lower distance cost. In addition, compared to other scenarios, fewer vehicles have been used in the AIMMS model. The figure below illustrates the comparison of all instances.

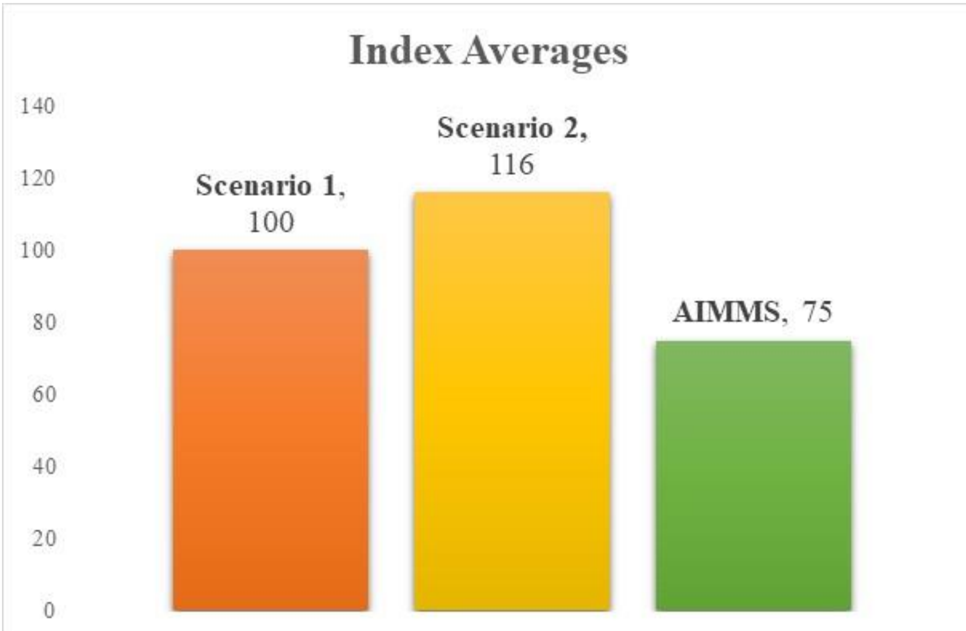

Figure 8: Index Averages

4.6 Discussion and Summary

Based on the results of computational experiences for all three scenarios of vehicle routing, we conclude Chapter four at the following points:

- Scenario 1 provides less distance cost in 1 sided round, but when combined for the complete round distances, it causes an increase in distance cost.
- Scenario 2 has more distance costs than scenario 1.
- AIMMS model computational cost is too high as compared to other models. So, it took a long time to provide the result.
- Due to high computational cost, the time constraint is only used for pickup and delivery.

- Since the current problem model is based on the exact approach and has been devised through linear programming. This is why the current problem model provides more efficient and effective results for the company than scenarios 1 and 2.

Chapter No. 5: Conclusion & Recommendations

The last chapter concludes the current research work. Subsequently, limitations and future recommendations are given for those aspects that were not taken into account in the present case study-based thesis research.

5.1 Conclusion

Thesis research contributes to the vehicle scheduling/routing problem. Where different variants of VRP have been taken up. From all the variants, mainly three variants have been incorporated in the research process, for which the necessary modeling has been developed through linear programming for the solution process. In the first step, we clearly described the vehicle scheduling/routing problem and its variants. After this, described the current research problem, its objective, constraints, and uniqueness of the current research problem. It is the problem in which multi depot vehicle scheduling problem has been taken with pickup and delivery along with the specification of the Dial-a-Ride problem. Further static demand for the orders is considered in the problem. The demand for the orders, order sequences and service time have been generated on a random basis. In addition, for distances data of the company's suppliers, distribution centers and depots taken from Royal Mail and represents through Bing map. The routing process of the current problem contributes to it as a combinatorial optimization problem (Paolo Toth, 2002).

In the current research, data collected from the company has been used to present optimized scheduling processes to meet the objective function. In this data overall 42 nodes have been considered (20 suppliers, 20 distribution centers and 2 depots). Based on the data, the objective function to minimize the distance cost has been modeled. In the current thesis, multi-depot VSP has presented and applied in the modeling process. Various components such as objective function, decision, and non-decisional variables and constraints have been formulated through linear programming. In addition, mixed integer programming has been applied to the mathematical formulation of the model.

Since the current research problem is known as combinatorial optimization. This is why it is accompanying with the property of NP-hardness. Therefore, Linear Programming-mixed integer programming has been run using software named AIMMS to effectively solve and analyze the problem. In addition, two scenarios have been developed and compared with the current model to test its effectiveness. While 30 data sets have been sampled for models and scenarios to analyze the robustness of the model. In the end, based on formulation, the solution has been explored for all samples which show the model effectiveness for the Multi depot vehicle scheduling problem with pickup and delivery. Which is represented by the index method. Based on the results, the following conclusions were drawn:

1. Due to the nature of the exact method – linear programming, it provides an optimal solution while minimizing the distance cost.
2. Due to the high level of computation cost, linear programming-mixed integer programming is very good with the small-scale vehicle scheduling problems.
3. Due to long processing time, linear programming-mixed integer programming is not suitable for large scale problems. Because it takes so long to deliver results that are less demanding in real life.
4. The two scenarios adopted in the comparison process to test the model's performance were less efficient. The first scenario was good for single orders but when the overall distance cost combined for all the orders, it may cause to increase the distance cost. While the

second scenario which has a limit of two orders were causing more distance costs. But this scenario was good for time limitation models. While time constraints were not part of the current research objective.

5. From a solution perspective, the AMMS model yields more reliable results at a lower distance cost than those two scenarios.
6. While performing the optimization, distance cost, time and use of vehicles are connected. Because if time constraints are included, distances and vehicles increase. But since there is no time constraint in the current research, so it doesn't affect the objective function of distance cost.

5.2 Research Limitations

In the current research thesis, multi depot vehicle scheduling problem with pickup and delivery model has been made which an objective function of distance cost minimization. Where in the end it proved its optimization effectiveness for an optimal solution of the real-life problem. In the research, the exact approach (linear programming-mixed integer programming) has been used for optimal results with a small data set. The model could be applied with the heuristics approach where one can derive the routing results for large instances. However, the thesis still has many questions that need to be answered.

Since the main area of research is based on modeling after theoretical contribution, so we have many limitations from a modeling perspective like lack of actual demand data, driving time limitations, traffic rules, etc. There are dynamic situations in real life that are difficult to solve. Meanwhile, planning through the system requires a high computational cost. In addition, some operational factors such as vehicle maintenance and charges are not considered in the thesis.

However, as a solution to the research model, the computational cost is high and it takes a long time to solve the problem where more rapid computations are needed to provide timely results. Moreover, we have taken the travel time for vehicle scheduling but due to time shortage, we didn't consider it to be an objective function.

5.3 Recommendations

There are various factors which can be better in the future for current research. Firstly, from a theoretical perspective, different variants of vehicle scheduling problems with pickup and delivery can be considered for the future like mixed VRPPD or simultaneously VRPPD. Further, time windows can be considered to extend current research. Our research has been done by using static demand, while this can be done through dynamic demand. Moreover, data for demand has been taken as a full load truck while it can be done by varying the demand requirements or by taking the real-time demand data.

Secondly from a modeling point of view, more complex settings can be made for the optimization of multi-depot vehicle scheduling problems with pickup and delivery. In addition, only 42 nodes of data have been taken with a sample of 30 instances. While in the future it can be done by taking more nodes, scenarios and a large sample instance. In the model, one node is served by one order, wherein in the future we can take more orders from one node. However, we can also incorporate time restrictions, to obtain optimal solutions from model for more real-life conditions. Vehicles and product types have been taken in the model are homogenous, while it can be done by using heterogeneous vehicles and products. From a solution point of view, different exact and heuristics methods can be used to analyze the problem, which can help in quality solutions.

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